

Boundary State Description of Tachyon Condensation

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Abstract

We construct the explicit boundary state description of the vortex-type (codimension two) tachyon condensation in brane-antibrane systems generalizing the known result of the kink-type (Frau et al. hep-th/9903123). In this description we show how the RR-charge of the lower dimensional D-branes emerges. We also investigate the tachyon condensation in T^4/\mathbf{Z}_2 orbifold and find that the twisted sector can be treated almost in the same way as the untwisted sector from the viewpoint of the boundary state. Further we discuss the higher codimension cases.

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1 Introduction

Recently there have been tremendous progresses in understanding non-BPS configurations of D-branes and tachyon condensations in them, pioneered by Sen (for a review see [1]). In Superstring theory most of these systems are realized as either brane-antibrane systems [2, 3, 4, 5] or non-BPS D-branes [6, 5, 7, 8, 9, 10, 11, 12, 13]. The open strings between a Dp-brane and an anti-Dp-brane are projected by the GSO projection opposite to the usual cases and the tachyon survives. A non-BPS D-brane is defined as a D-brane without any GSO projections and the tachyonic instability also occurs. Sen argued that if the condensation of the constant tachyon field stabilizes the system, then the system will finally go down to the vacuum [4] and if the condensation has nontrivial configurations such as kinks or vortexes, then the final object will be D-branes of corresponding codimension [3, 5, 8, 14]. For example, the kink configuration in $Dp - \overline{Dp}$ brane system is identified as a non-BPS $D(p - 1)$ -brane.

Three different approaches have been considered ⁴ to analyze those systems. The first one is to use conformal field theory descriptions of string world sheet with a boundary [5, 8, 9, 15, 14]. In this method, the tachyon condensation can be regarded as a marginal deformation of the boundary conformal field theory (BCFT) at a special radius. The second is the K-theory approach by Witten. He argued that the topological charges of lower D-branes in a non-BPS configuration of D-branes are classified by the corresponding K-group [17, 12]. In other words we can say that the topological configurations of tachyon fields one-to-one correspond to the element of the K-group. The third one is the string field theory description [18]. The tachyon potential has been calculated in the case of a D-brane in bosonic string [19] and a non-BPS D-brane in Superstring [20]. The numerical results are in good agreement with the Sen's conjecture that the system at the minimum of the tachyon potential can be identified as the vacuum. The generations of lower dimensional D-brane charges have been also discussed in this formalism [21].

In this paper we are interested in the first approach. From the viewpoint of the open strings the BCFT descriptions of tachyon kink condensations have been given in [5, 8, 9] for a brane-antibrane system or a non-BPS D-brane in the presence of the orbifold and orientation projection. In order to discuss the generation of codimension two D-branes, the vortex line configuration of the tachyon field is needed and is realized in [15] as a pair of the vortex and anti-vortex.

On the other hand we can use the boundary state formalism (for example see [22]), which can give more systematic CFT description of D-branes. In this formalism D-branes are constructed in the closed string Hilbert space. Therefore the couplings of D-branes to NSNS, RR-fields can be written down explicitly. The equivalence between the open

⁴Quite recently a new approach which utilizes the noncommutative field theory description of the world volume theory has been considered in the presence of a large B-field [16].

string description and the closed string description should be required as in the usual BPS D-branes and this crucial constraint is called Cardy's condition [23]. The boundary state description of the tachyon kink pair in flat space was first constructed in [24]. Also in the case of the bosonic string the boundary state description was discussed in [25]. In the first half of this paper we extend this construction [24] to the case of tachyon vortex pairs in flat space. We construct explicitly the boundary state which describes the tachyon vortex pair condensation in $D2 - \overline{D2}$ system at the critical radius. The Cardy's condition is established and the emergence of lower D-brane RR-charges is shown explicitly. At the point where the tachyon condensation is maximum the boundary state of the system is identified as that of a $D0 - \overline{D0}$ system. Many other points correspond to the bound state of $D2 - \overline{D2}$ and $D0 - \overline{D0}$. Also the requirement⁵ of the nontrivial "Chan-Paton factor of closed string" is verified in this formalism. Further we generalize these results into the higher codimension cases.

In the latter half we treat the case of T^4/\mathbf{Z}_2 orbifold. Remarkably it is shown that the twisted sector boundary state can be written as the untwisted sector boundary state of another fields on the world sheet at the critical radius by performing bosonizations and fermionizations. Using this fact we can describe the tachyon kink condense explicitly starting from a $D0 - \overline{D0}$ system and show that the untwisted RR charge vanishes and the twisted RR charge remains after the condensation, verifying the known identification [8] of the final object as a non-BPS D1-brane between the fixed points. This boundary state approach enables us to generalize the tachyon kink in the orbifold theory into the higher codimension cases such as the decay mode from $D4 - \overline{D4}$ to $D0 - \overline{D0}$, which has not been discussed before. We also discuss the relation between the bose-fermi degeneracy [26, 27] and the "bosonization procedure" in the boundary state description.

The paper is organized as follows. In section 2 we review the known results about the BCFT description of a tachyon vortex pair in a $D2 - \overline{D2}$ system and a tachyon kink pair in a orbifolded $D0 - \overline{D0}$ system. Further we investigate some details. In section 3 we construct the explicit boundary state which describes the condensation of the tachyon vortex pair in a $D2 - \overline{D2}$ system. We show the final object can be identified with $D0 - \overline{D0}$. Next we generalize the results and see that the tachyon condensation generates D-brane charges of higher codimension. In section 4 we study the tachyon kink in T^4/\mathbf{Z}_2 orbifold. We give the corresponding boundary state and identify the final object. We also generalize the result into higher codimension cases. In section 5 we summarize the results and draw conclusions. In appendix A we give a breif review of boundary state and show our conventions. In appendix B we prove that the "bosonized" boundary state indeed satisfies the original boundary condition including detailed cocycle factors.

⁵In the case of a non-BPS D-brane the similar requirement was mentioned in [7, 1].

2 CFT description of tachyon condensation

In this section we review the descriptions of tachyon condensation from the viewpoint of open strings. These results are needed when we compare the results with those gained in the boundary state formalism. First we see the vortex-type tachyon condensation in the $D2 - \overline{D2}$ system following [15, 5] and investigate some details in a slightly generalized situation. Next we review several known facts about the brane-antibrane system in T^4/\mathbf{Z}_2 orbifold and the tachyon condensation in that system. Such a system was first discussed in [8, 11] and also considered in [9] using the T-dualized picture. In this paper we consider type II string theory only in the weak coupling region.

2.1 Tachyon condensation in a $D2 - \overline{D2}$ system

We take a parallel $D2$ -brane and an anti $D2$ -brane in type IIA string theory along x^1, x^2 and compactify these directions on a torus of radii ⁶ $R_1 = 1, R_2 = 1$. Then we set a \mathbf{Z}_2 Wilson line along each circle. There are four types of Chan-Paton factors for the open strings in $D2 - \overline{D2}$ system and are denoted by $1, \sigma_1, \sigma_2, \sigma_3$ using Pauli matrices. We use $1, \sigma_3$ in order to represent the open strings with both ends on the same brane and the spectrum is determined by the conventional GSO projection. On the other hand σ_1, σ_2 correspond to the open strings with two ends on two different branes and follow the opposite GSO projection allowing the tachyon in the spectrum.

We consider the condensation of the following two types⁷ of the tachyon field

$$T_{(1)}(x^1, x^2) = e^{i\frac{1}{2}(x^1+x^2)} - e^{-i\frac{1}{2}(x^1+x^2)}, \quad (2.1)$$

$$T_{(2)}(x^1, x^2) = -ie^{i\frac{1}{2}(x^1-x^2)} + ie^{-i\frac{1}{2}(x^1-x^2)}. \quad (2.2)$$

If we switch on only one of these, we get the tachyon kink configuration and a codimension one D-brane or a non-BPS D1-brane will be generated. On the other hand if we condense both at the same time, the tachyon vortex line pair configuration will lead to a pair of codimension two D-branes or a $D0 - \overline{D0}$ system.

The corresponding open string vertex operators in (0)-picture are written as

$$\begin{aligned} V_{T1} &= (\chi^1 + \chi^2)(e^{i\frac{1}{2}(X^1+X^2)} + e^{-i\frac{1}{2}(X^1+X^2)}) \otimes \sigma_1, \\ V_{T2} &= (\chi^2 - \chi^1)(e^{i\frac{1}{2}(X^1-X^2)} + e^{-i\frac{1}{2}(X^1-X^2)}) \otimes \sigma_2, \end{aligned} \quad (2.3)$$

where $X^i = X_R^i + X_L^i$, $\chi^i = \chi_R^i + \chi_L^i$ ($i = 1, 2$) denote the bosonic fields on the string world sheet in NS-R formalism and their superpartners.

⁶In this paper we use $\alpha' = 1$ unit.

⁷There are also other two marginal deformations which represent other tachyon condensations. But these correspond to the shift of the vortex line center and the physical phenomena which occurs by such tachyon condensations do not change if we ignore these. Thus we only consider the tachyon fields (2.1), (2.2) below.

Notice that at the radii $R_1 = 1$, $R_2 = 1$ (critical radii) the lightest tachyon vertex operators become marginal owing to the Wilson lines and the tachyon condensation corresponding to such operators can be treated as the marginal deformation of CFT.

Now let us rotate the coordinates by $\frac{\pi}{4}$

$$\begin{aligned} Y^1 &= \frac{1}{\sqrt{2}}(X^1 + X^2), & Y^2 &= \frac{1}{\sqrt{2}}(X^1 - X^2), \\ \psi^1 &= \frac{1}{\sqrt{2}}(\chi^1 + \chi^2), & \psi^2 &= \frac{1}{\sqrt{2}}(\chi^1 - \chi^2). \end{aligned} \quad (2.4)$$

This procedure enables us to use the method of bosonization and fermionization as follows

$$\begin{aligned} e^{i\sqrt{2}Y_R^i} &= \frac{1}{\sqrt{2}}(\xi_R^i + i\eta_R^i) \otimes \tau_i, & e^{i\sqrt{2}Y_L^i} &= \frac{1}{\sqrt{2}}(\xi_L^i + i\eta_L^i) \otimes \tau_i, \\ e^{i\sqrt{2}\phi_R^i} &= \frac{1}{\sqrt{2}}(\xi_R^i + i\psi_R^i) \otimes \tilde{\tau}_i, & e^{i\sqrt{2}\phi_L^i} &= \frac{1}{\sqrt{2}}(\xi_L^i + i\psi_L^i) \otimes \tilde{\tau}_i, \\ e^{i\sqrt{2}\phi_R^i} &= \frac{1}{\sqrt{2}}(\eta_R^i + i\psi_R^i) \otimes \hat{\tau}_i, & e^{i\sqrt{2}\phi_L^i} &= \frac{1}{\sqrt{2}}(\eta_L^i + i\psi_L^i) \otimes \hat{\tau}_i, \end{aligned} \quad (2.5)$$

where $\tau_i, \tilde{\tau}_i, \hat{\tau}_i (i = 1, 2)$ are cocycle factors [15, 8] and we also assume $\psi_{L,R}^i, \eta_{L,R}^i$ have the cocycle factor $\tau_3, \tilde{\tau}_3$. To be exact, other kinds of cocycle factors are needed in front of the exponential fields. The latter type of cocycle factors, which we will call second-type cocycle factors below, can not be ignored when we later discuss the bosonizations and fermionizations of boundary states. We leave the details in the appendix B.

The operator product expansions (OPE) among these fields are⁸

$$\begin{aligned} Y_R^i(z)Y_R^j(0) &\sim -\frac{1}{2}\delta_{ij}\ln z, & Y_L^i(\bar{z})Y_L^j(0) &\sim -\frac{1}{2}\delta_{ij}\ln \bar{z}, \\ \xi_R^i(z)\xi_R^j(0) &\sim \delta_{ij}\frac{i}{z}, & \xi_L^i(\bar{z})\xi_L^j(0) &\sim -\delta_{ij}\frac{i}{\bar{z}}, \\ \eta_R^i(z)\eta_R^j(0) &\sim \delta_{ij}\frac{i}{z}, & \eta_L^i(\bar{z})\eta_L^j(0) &\sim -\delta_{ij}\frac{i}{\bar{z}}. \end{aligned} \quad (2.6)$$

Also the following identities are useful:

$$\begin{aligned} \eta_R^i\xi_R^i &= i\sqrt{2}\partial Y_R^i, & \eta_L^i\xi_L^i &= -i\sqrt{2}\bar{\partial} Y_L^i, \\ \psi_R^i\xi_R^i &= i\sqrt{2}\partial\phi_R^i, & \psi_L^i\xi_L^i &= -i\sqrt{2}\bar{\partial}\phi_L^i. \end{aligned} \quad (2.7)$$

Now we can express the tachyon vertex operators (2.3) in the following convenient way

$$\begin{aligned} V_{T1} &= 2i\psi^1\xi^1 \otimes \tau_2 \otimes \sigma_1 = -2\sqrt{2}\partial\phi^1 \otimes \tau_2 \otimes \sigma_1, \\ V_{T2} &= -2i\psi^2\xi^2 \otimes \tau_1 \otimes \sigma_2 = +2\sqrt{2}\bar{\partial}\phi^2 \otimes \tau_1 \otimes \sigma_2, \end{aligned} \quad (2.8)$$

⁸Note that the factors i in the bosonic field OPE's are due to second-type cocycle factors.

where ∂ denotes tangential derivative along the boundary. Then the tachyon condensation is represented as the insertion of the following Wilson lines in terms of the field ϕ

$$\exp \left(\frac{i\alpha}{2\sqrt{2}} \oint \partial\phi^1 \otimes \tau_2 \otimes \sigma_1 + \frac{i\beta}{2\sqrt{2}} \oint \partial\phi^2 \otimes \tau_1 \otimes \sigma_2 \right), \quad (2.9)$$

where \oint denotes integration along the boundary and α, β mean parameters of tachyon condensations. Notice that $\tau_2 \otimes \sigma_1$ commutes with $\tau_1 \otimes \sigma_2$ and the above Wilson line is well defined without path ordering. The open string spectrum in the R sector does not change in the presence of the Wilson line because for the R sector ϕ satisfies Neumann boundary condition at one end and Dirichlet boundary condition at the other end and there is no zero mode for ϕ [5]. Therefore we will investigate only the NS sector.

Now let us define several projection operators in the following way

$$\begin{aligned} (-1)^F &: |0\rangle \rightarrow -|0\rangle, \quad \psi^i \rightarrow -\psi^i, \quad (\xi^i, \eta^i) \rightarrow (\xi^i, \eta^i), \\ h_1 &: (\xi^1, \eta^1) \rightarrow -(\xi^1, \eta^1), \quad (\xi^2, \eta^2) \rightarrow (\xi^2, \eta^2), \quad \psi^i \rightarrow \psi^i, \\ &\quad Y_{L,R}^1 \rightarrow Y_{L,R}^1 + \frac{\pi}{\sqrt{2}}, \quad Y_{L,R}^2 \rightarrow Y_{L,R}^2, \\ h_2 &: (\xi^1, \eta^1) \rightarrow (\xi^1, \eta^1), \quad (\xi^2, \eta^2) \rightarrow -(\xi^2, \eta^2), \quad \psi^i \rightarrow \psi^i, \\ &\quad Y_{L,R}^1 \rightarrow Y_{L,R}^1, \quad Y_{L,R}^2 \rightarrow Y_{L,R}^2 + \frac{\pi}{\sqrt{2}}. \end{aligned} \quad (2.10)$$

As is clear from the above definition, $(-1)^F$ is the fermion number on the world sheet and h_1, h_2 are the translation operators in the direction of Y^1, Y^2 . We also define $(-1)^{F^\phi}, h_1^\phi, h_2^\phi$ similarly for ϕ .

Since so far we have implicitly assumed the radius of circle in the direction of Y^1, Y^2 is $\sqrt{2}$, we should have a certain constraint in order to realize the physical periodicity $X^1 \sim X^1 + 2\pi, X^2 \sim X^2 + 2\pi$ taking the effect of the Wilson line into consideration. Such a constraint is given as

$$(-1)^F h_1 h_2 = (-1)^{F^\phi} h_1^\phi h_2^\phi = 1, \quad (2.11)$$

where we used eq.(2.10). There are eight sectors in NS sector which survive this projection as follows

$$\begin{aligned} &1 \otimes 1, \quad 1 \otimes \tau_3, \quad \sigma_3 \otimes 1, \quad \sigma_3 \otimes \tau_3, \\ &\sigma_1 \otimes \tau_1, \quad \sigma_1 \otimes \tau_2, \quad \sigma_2 \otimes \tau_1, \quad \sigma_2 \otimes \tau_2. \end{aligned} \quad (2.12)$$

Four of these are insensitive to the tachyon condense or equally the insertion of the Wilson lines. But the momenta of ϕ in the other four sectors are shifted in proportion to the deformation parameters α, β . The details are shown in Table 1. Note that α, β have periodicity $\alpha \sim \alpha + 2, \beta \sim \beta + 2$ by applying the same argument discussed in [5].

The main claim in [15] is that if the tachyon condensation develops into the point $\alpha = 1, \beta = 1$, then the system is identified as the $D0 - \overline{D0}$ system where $D0$ -brane

Table 1: Momentum shifts due to the tachyon condensation. Here we introduced four coefficients a, b, c, d of a given vertex V as $V = a(1 \otimes \tau_3) + b(\sigma_1 \otimes \tau_1) + c(\sigma_3 \otimes 1) + d(\sigma_2 \otimes \tau_2)$.

sector	a	b	c	d	ΔP_{ϕ^1}	ΔP_{ϕ^2}
$1 \otimes \tau_3$	1	i	1	$-i$	$\frac{\alpha}{\sqrt{2}}$	$\frac{\beta}{\sqrt{2}}$
$\sigma_1 \otimes \tau_1$	1	i	-1	i	$\frac{\alpha}{\sqrt{2}}$	$-\frac{\beta}{\sqrt{2}}$
$\sigma_3 \otimes 1$	1	$-i$	-1	$-i$	$-\frac{\alpha}{\sqrt{2}}$	$\frac{\beta}{\sqrt{2}}$
$\sigma_2 \otimes \tau_2$	1	$-i$	1	i	$-\frac{\alpha}{\sqrt{2}}$	$-\frac{\beta}{\sqrt{2}}$

and $\overline{D0}$ -brane sit at $(x^1, x^2) = (0, 0)$ and $(x^1, x^2) = (\pi, \pi)$. For example the open string spectrum at $\alpha = 0, \beta = 0$ is shown to be the same as the spectrum at $\alpha = 1, \beta = 1$ because the momentum shift is $\Delta P_{\phi^1} = \pm \frac{1}{\sqrt{2}}$, $\Delta P_{\phi^2} = \pm \frac{1}{\sqrt{2}}$ and for each state the value of $(-1)^F h_1 h_2$ does not change. But in order to prove the claim it is necessary to distinguish $D0 - \overline{D0}$ from its T-dual equivalent $D2 - \overline{D2}$ at the self-dual radii $R_1 = R_2 = 1$ and explain⁹ the emergence of NSNS and RR-charge corresponding to $D0 - \overline{D0}$. For these purposes the boundary state description which will be discussed in the following sections is very useful and systematic since the D-branes are represented in the closed string Hilbert space in this description.

2.2 Tachyon condensation in T^4/\mathbf{Z}_2 orbifold

Let us denote¹⁰ y^6, y^7, y^8, y^9 as the coordinates of T^4/\mathbf{Z}_2 with the involution $I_4 : (y^6, y^7, y^8, y^9) \rightarrow (-y^6, -y^7, -y^8, -y^9)$ and assume the radii of the torus are given as $R^6 = \sqrt{2}, R^7 = R^8 = R^9 = R$.

First we consider a fractional $D0 - \overline{D0}$ system where a $D0$ -brane is sitting on a fixed point $(y^6, y^7, y^8, y^9) = (0, 0, 0, 0)$ and a $\overline{D0}$ -brane is sitting on another fixed point $(y^6, y^7, y^8, y^9) = (\pi\sqrt{2}, 0, 0, 0)$. Each of them has $\frac{1}{2}$ tension and $\frac{1}{2}$ RR charge of a bulk $D0$ -brane and can be interpreted as a $D2$ -brane wrapped on the vanishing 2-cycle which corresponds to the fixed point [28].

Such a system has no tachyonic modes which survive the I_4 projection and therefore is stable at the critical radius. One of the marginal “tachyon” vertex in (0)-picture which represents a tachyon kink in the y^6 direction is given as

$$V_T = \psi^6 (e^{i\frac{1}{\sqrt{2}}(Y_R^6 - Y_L^6)} \pm e^{-i\frac{1}{\sqrt{2}}(Y_R^6 - Y_L^6)}) \otimes \sigma_1, \quad (2.13)$$

⁹For the NSNS-charge the explanation is given in [15] perturbatively by considering a certain disk amplitude where a NSNS vertex operator was inserted.

¹⁰Here we have used not x but y because later we will identify these coordinates as $\frac{\pi}{4}$ rotated ones.

where \pm depend on the relative twisted sector charge of the system or equally the projection $I_4 = \pm 1$ and below we only consider the case of $+$ sign. Using the bosonizations and fermionizations (2.5), it is easy to see $V_T \propto \partial\phi^6 \otimes \sigma_1$ and the tachyon condensation¹¹ is described by the following Wilson line¹²

$$W = \exp \left(i \frac{\alpha}{2\sqrt{2}} \oint \partial\phi^6 \otimes \sigma_1 \right). \quad (2.14)$$

If we condense the tachyon into $\alpha = 1$, then the system is identified with the non-BPS D1-brane stretching between the fixed points. The justification of this statement will be given later by constructing the boundary state. In [8] this non-BPS D1-brane is identified with a D2-brane wrapped on a non-supersymmetric cycle. Later we will also construct the marginal deformation from $D4 - \overline{D4}$ to $D0 - \overline{D0}$ in this orbifold theory.

There is also a known interesting fact. If we consider a non-BPS D1-brane at the special radius $R = \frac{1}{\sqrt{2}}$, then the vacuum amplitude of the system vanishes and the system develops the bose-fermi degeneracy [26]. Later this phenomenon will be discussed in terms of the boundary state description.

3 Boundary state description of tachyon condensation

In this section, we construct the boundary state for a $D2 - \overline{D2}$ system and condensate a tachyon vortex pair. Mainly we follow the line of [24], where a tachyon kink was considered. The crucial difference from that case is the emergence of nontrivial ‘‘Chan-Paton factors’’ in closed string sectors. After the condensation the final object is identified with a $D0 - \overline{D0}$ system as expected. Next we also calculate the vacuum amplitude and investigate the consistency with open string picture. Finally we generalize these results into the higher codimension cases. The definition and brief review of boundary states are given in appendix A.

¹¹Note that this marginal deformation is just the opposite to that considered in [8], where the deformation from non-BPS D1-brane to $D0 - \overline{D0}$ is considered.

¹²If we consider the case of the opposite twisted charge, then we get $V_T \propto \partial\phi'^6 \otimes \sigma_1$ and $W = \exp \left(i \frac{\alpha}{2\sqrt{2}} \oint \partial_n \phi'^6 \otimes \sigma_1 \right)$, where ∂_n denotes derivative in the normal direction.

3.1 Bosonization of the boundary state and tachyon condensation

First the boundary state for a D2-brane at x^i ($i = 3 \sim 9$) which is extended to x^1, x^2 without any Wilson lines is given as follows :

$$\begin{aligned}
|D2, x^i\rangle &= \frac{T_{p=2}}{2} (|D2, x^i\rangle_{NSNS} + |D2, x^i\rangle_{RR}), \\
|D2, x^i\rangle_{NSNS} &= \frac{1}{2} \int \left(\frac{dk}{2\pi} \right)^7 e^{ikx} [|D2, +, k_X^i\rangle_{NSNS} - |D2, -, k_X^i\rangle_{NSNS}], \\
|D2, x^i\rangle_{RR} &= 2 \int \left(\frac{dk}{2\pi} \right)^7 e^{ikx} [|D2, +, k_X^i\rangle_{RR} + |D2, -, k_X^i\rangle_{RR}],
\end{aligned} \tag{3.1}$$

where $T_p = 2^{3-p} \pi^{\frac{7}{2}-p}$ is the normalization¹³ of the Dp-brane boundary state and k^i ($i = 3 \sim 9$) are the momenta in the direction of x^i . The explicit forms of $|D2, \gamma, k^i\rangle_{sector}$ are given below. The NSNS-sector is

$$\begin{aligned}
|D2, \gamma, k_X^i\rangle_{NSNS} &= \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \{ (\alpha_X)^0_{-n} (\tilde{\alpha}_X)^0_{-n} + \sum_{j=3}^7 (\alpha_X)^j_{-n} (\tilde{\alpha}_X)^j_{-n} \} \right] \\
&\times \exp \left[-i\gamma \sum_{n=1}^{\infty} \{ \chi_{-n+\frac{1}{2}}^0 \tilde{\chi}_{-n+\frac{1}{2}}^0 + \sum_{j=3}^7 \chi_{-n+\frac{1}{2}}^j \tilde{\chi}_{-n+\frac{1}{2}}^j \} \right] \\
&\times \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_X)^i_{-n} (\tilde{\alpha}_X)^i_{-n} \right] \exp \left[+i\gamma \sum_{n=1}^{\infty} \sum_{i=1}^2 \chi_{-n+\frac{1}{2}}^i \tilde{\chi}_{-n+\frac{1}{2}}^i \right] \\
&\times |D2, \gamma, k_X^i\rangle_{NSNS}^{(0)},
\end{aligned} \tag{3.2}$$

and zero-mode

$$|D2, \gamma, k_X^i\rangle_{NSNS}^{(0)} = \sum_{\vec{w}_X \in \mathbf{Z}^2} |\vec{0}, \vec{w}_X\rangle \otimes |k_X^i\rangle \otimes |\Omega\rangle_{NSNS}^{(0)}, \tag{3.3}$$

where $|\Omega\rangle_{NSNS}^{(0)}$ is the vacuum of world sheet theory and $|\vec{n}_X, \vec{w}_X\rangle$ represents the zero mode part of T^2 which has momenta $\vec{n}_X = (n_X^1, n_X^2)$ and windings $\vec{w}_X = (w_X^1, w_X^2)$. The RR-sector is given as

$$\begin{aligned}
|D2, \gamma, k_X^i\rangle_{RR} &= \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \{ (\alpha_X)^0_{-n} (\tilde{\alpha}_X)^0_{-n} + \sum_{j=3}^7 (\alpha_X)^j_{-n} (\tilde{\alpha}_X)^j_{-n} \} \right] \\
&\times \exp \left[-i\gamma \sum_{n=1}^{\infty} \{ \chi_{-n}^0 \tilde{\chi}_{-n}^0 + \sum_{j=3}^7 \chi_{-n}^j \tilde{\chi}_{-n}^j \} \right] \\
&\times \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_X)^i_{-n} (\tilde{\alpha}_X)^i_{-n} \right] \exp \left[+i\gamma \sum_{n=1}^{\infty} \sum_{i=1}^2 \chi_{-n}^i \tilde{\chi}_{-n}^i \right]
\end{aligned}$$

¹³This can be determined by computing the cylinder amplitude.

$$\times |D2, \gamma, k_X^i \rangle_{RR}^{(0)}, \quad (3.4)$$

and zero-mode

$$|D2, \gamma, k_X^i \rangle_{RR}^{(0)} = \sum_{\vec{w}_X \in \mathbb{Z}^2} |\vec{0}, \vec{w}_X \rangle \otimes |k_X^i \rangle \otimes |\Omega, \gamma \rangle_{RR}^{(0)}, \quad (3.5)$$

where $|\Omega, \gamma \rangle_{RR}^{(0)}$ is the solution to

$$\{\chi_0^\mu - i\gamma\chi_0^\mu\} |\Omega, \gamma \rangle_{RR}^{(0)} = \{\chi_0^i + i\gamma\chi_0^i\} |\Omega, \gamma \rangle_{RR}^{(0)} = 0 \quad (\mu = 0, 1, 2). \quad (3.6)$$

The normalizations of the zero modes are defined as

$$\begin{aligned} \langle \Omega | \Omega \rangle_{NSNS}^{(0)} &= 1, & \langle \Omega, \gamma | \Omega, \gamma' \rangle_{RR}^{(0)} &= \delta_{\gamma, \gamma'}, \\ \langle \vec{n}_X, \vec{w}_X | \vec{n}'_X, \vec{w}'_X \rangle &= V \delta_{n, n'} \delta_{w, w'}, & \langle k_X^i | k_X^i \rangle &= (2\pi)^7 \delta^7(k - k'), \end{aligned} \quad (3.7)$$

where V is the volume of the time direction. Note that this is the solution to the condition (A.4) for a boundary state of a D2-brane. We use $\gamma = \pm 1$ to indicate each choice of the open string boundary condition (A.4). Also note that since we have used the light cone formalism [29, 13], the superscripts of oscillators run from 0 to 7, not to 9. We have divided the oscillator parts into $X^0, X^3 \sim X^7$ and X^1, X^2 (χ^μ are also divided) in eq(3.2), (3.4). This is because $X^0, X^3 \sim X^7$ part does not contribute to the later calculations of tachyon condensation importantly. Therefore we abbreviate $X^0, X^3 \sim X^7, \chi^0, \chi^3 \sim \chi^7$ part and k^i from now on.

Next, we construct the boundary state for $D2 - \overline{D2}$ system where the position of the D2-brane and the $\overline{D2}$ -brane is $x^i = 0$. This is given by the superposition of the boundary states for a D2-brane as

$$\begin{cases} |D2 - \overline{D2}, \gamma \rangle_{NSNS} &= |D2, \gamma \rangle_{NSNS} + |D2', \gamma \rangle_{NSNS}, \\ |D2 - \overline{D2}, \gamma \rangle_{RR} &= |D2, \gamma \rangle_{RR} - |D2', \gamma \rangle_{RR}. \end{cases} \quad (3.8)$$

Here, we have two important points. One point is that we have switched on \mathbf{Z}_2 Wilson lines of the second D2-brane and we have expressed such a boundary state as that with a prime. Next point is the second D2-brane is the anti D-brane. Since an anti D-brane has the opposite RR charge to a D-brane, we have added a minus sign to the second boundary state of RR sector.

From eq.(3.8), the oscillator parts of $|D2 - \overline{D2}, \gamma \rangle_{NSNS, RR}$ are the same as eq.(3.2), eq.(3.4), and the zero mode parts are given by

$$\begin{aligned} |D2 - \overline{D2}, \gamma \rangle_{NSNS}^{(0)} &= \sum_{\vec{w}_X \in \mathbb{Z}^2} |\vec{0}, \vec{w}_X \rangle_{NSNS}^{(0)} + (-1)^{w_X^1 + w_X^2} |\vec{0}, \vec{w}_X \rangle_{NSNS}^{(0)}, \\ |D2 - \overline{D2}, \gamma \rangle_{RR}^{(0)} &= \sum_{\vec{w}_X \in \mathbb{Z}^2} |\vec{0}, \vec{w}_X, \gamma \rangle_{RR}^{(0)} - (-1)^{w_X^1 + w_X^2} |\vec{0}, \vec{w}_X, \gamma \rangle_{RR}^{(0)}, \end{aligned} \quad (3.9)$$

where the phase factors $(-1)^{w_X^1}, (-1)^{w_X^2}$ are due to \mathbf{Z}_2 Wilson lines [3, 24].

Then let us perform $\frac{\pi}{4}$ rotation (2.4) and rewrite¹⁴ the boundary state $|D2 - \overline{D2}, \gamma\rangle_{NSNS, RR}$ by using the fields $(Y^1, Y^2, \psi^1, \psi^2)$ as follows

$$\begin{aligned} |D2 - \overline{D2}, \gamma\rangle_{NSNS} &= \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_Y)^i_{-n} (\tilde{\alpha}_Y)^i_{-n} \right] \exp \left[+ i \gamma \sum_{n=1}^{\infty} \sum_{i=1}^2 \psi_{-n+\frac{1}{2}}^i \tilde{\psi}_{-n+\frac{1}{2}}^i \right] \\ &\times 2 \sum_{\vec{w}_Y \in \mathbf{Z}^2} |\vec{0}, 2\vec{w}_Y\rangle_{NSNS}^{(0)}, \end{aligned} \quad (3.10)$$

$$\begin{aligned} |D2 - \overline{D2}, \gamma\rangle_{RR} &= \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_Y)^i_{-n} (\tilde{\alpha}_Y)^i_{-n} \right] \exp \left[+ i \gamma \sum_{n=1}^{\infty} \sum_{i=1}^2 \psi_{-n}^i \tilde{\psi}_{-n}^i \right] \\ &\times 2 \sum_{\vec{w}_Y \in \mathbf{Z}^2} |\vec{0}, 2\vec{w}_Y + \vec{1}, \gamma\rangle_{RR}^{(0)}, \end{aligned} \quad (3.11)$$

where we defined $\vec{1} = (1, 1)$.

As explained in section 2, we have to change the base of the world sheet fields (Y, ψ) into the bosonized ones (ϕ, η) in order to describe the tachyon condensation. Then by using (ϕ, η) modes how are the boundary states (3.10), (3.11) represented? Generalizing the discussion in [24], we argue that the following boundary states are equivalent to eq.(3.10) and (3.11) respectively for $\gamma = +1$:

$$\begin{aligned} |D2 - \overline{D2}, +\rangle_{NSNS} &= 2 \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} \phi_{-n}^i \tilde{\phi}_{-n}^i \right] \exp \left[+ i \sum_{n=1}^{\infty} \sum_{i=1}^2 \eta_{-n+\frac{1}{2}}^i \tilde{\eta}_{-n+\frac{1}{2}}^i \right] \\ &\times \sum_{\vec{w}_\phi \in \mathbf{Z}^2} |\vec{0}, 2\vec{w}_\phi\rangle_{NSNS}^{(0)}, \end{aligned} \quad (3.12)$$

$$\begin{aligned} |D2 - \overline{D2}, +\rangle_{RR} &= 2 \exp \left[- \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} \phi_{-n}^i \tilde{\phi}_{-n}^i \right] \exp \left[+ i \sum_{n=1}^{\infty} \sum_{i=1}^2 \eta_{-n}^i \tilde{\eta}_{-n}^i \right] \\ &\times \sum_{\vec{w}_\phi \in \mathbf{Z}^2} |\vec{0}, 2\vec{w}_\phi + \vec{1}, +\rangle_{RR}^{(0)}. \end{aligned} \quad (3.13)$$

These are obtained simply by replacing $(\alpha_Y)^i_n, \psi_n^i, w_x^i$ in eq.(3.10), (3.11) with $\phi_n^i, \eta_n^i, w_\phi^i$. In the appendix B we show they indeed satisfy the desirable boundary conditions if we take the detailed (second-type) cocycle factors into consideration :

$$\partial_2 Y^i(w, \bar{w})|_{\sigma_2=0} |D2 - \overline{D2}, +\rangle_{NSNS, RR} = 0, \quad (3.14)$$

$$(\psi_R^i(w) - i\psi_L^i(\bar{w}))|_{\sigma_2=0} |D2 - \overline{D2}, +\rangle_{NSNS, RR} = 0. \quad (3.15)$$

Note that eq.(3.14) and (3.15) are not enough¹⁵ for the proof of the equivalence. But further we can see that for several closed string states eq.(3.12) and (3.13) have the same

¹⁴Note that here we have regarded the radii of Y^1, Y^2 direction as $R^1 = R^2 = \frac{1}{\sqrt{2}}$ and we have used the relations $w_Y^1 = w_X^1 + w_X^2, w_Y^2 = w_X^1 - w_X^2$.

¹⁵This is because these constraints do not determine the detailed structures of the zero modes such as Wilson lines.

overlap as eq.(3.10) and (3.11) do ¹⁶. The third evidence is the equality of their partition functions, which we will see in the next subsection. We propose that these three evidences are enough for the proof of the equivalence.

On the other hand, $|D2 - \overline{D2}, - \rangle_{NSNS,RR}$ is given by acting left-moving fermion number operator $(-1)^{\tilde{F}_Y}$ of (Y, ψ) system :

$$|D2 - \overline{D2}, - \rangle_{NSNS,RR} = (-1)^{\tilde{F}_Y} |D2 - \overline{D2}, + \rangle_{NSNS,RR}. \quad (3.16)$$

Note that the action of $(-1)^{\tilde{F}_Y}$ is given as

$$\begin{aligned} (-1)^{\tilde{F}_Y} : \quad & (\tilde{\alpha}_Y)_n^i \rightarrow (\tilde{\alpha}_Y)_n^i, \quad \tilde{\psi}_n^i \rightarrow -\tilde{\psi}_n^i, \\ & \tilde{\phi}_n^i \rightarrow -\tilde{\phi}_n^i, \quad \tilde{\eta}_n^i \rightarrow \tilde{\eta}_n^i, \end{aligned} \quad (3.17)$$

and to zero mode

$$(-1)^{\tilde{F}_Y} : |\vec{n}_\phi, \vec{w}_\phi \rangle \rightarrow (\text{phase}) \times \left| \frac{\vec{w}_\phi}{2}, 2\vec{n}_\phi \right\rangle, \quad (3.18)$$

where "phase" comes from cocycle factors. Therefore for example, $|D2 - \overline{D2}, - \rangle_{NS}$ is given by

$$\begin{aligned} |D2 - \overline{D2}, - \rangle_{NSNS} = & 2 \exp \left[+ \sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} \phi_{-n}^i \tilde{\phi}_{-n}^i \right] \exp \left[+ i \sum_{n=1}^{\infty} \sum_{i=1}^2 \eta_{-n+\frac{1}{2}}^i \tilde{\eta}_{-n+\frac{1}{2}}^i \right] \\ & \times \sum_{\vec{w}_\phi \in \mathbf{Z}^2} (-1)^{w_\phi^1 + w_\phi^2} |\vec{w}_\phi, \vec{0} \rangle_{NSNS}^{(0)}. \end{aligned} \quad (3.19)$$

Then it is straightforward to condense the tachyon by using the Wilson lines (2.9). Since closed strings usually do not have Chan-Paton factors, the tachyon condensation in the closed string viewpoint corresponds to the insertion of the trace of (2.9) in front of the boundary state and the trace is given as

$$W_1(\alpha, \beta) \propto \cos \left(\frac{\pi \alpha w_\phi^1}{2} \right) \cos \left(\frac{\pi \beta w_\phi^2}{2} \right). \quad (3.20)$$

But, this is not sufficient. In [30] the authors argued that the Wess-Zumino terms in the effective action of $D2 - \overline{D2}$ systems should possess the following coupling.

$$\int C_1 \wedge dT \wedge d\bar{T}, \quad (3.21)$$

where T, \bar{T} are complex tachyon fields, and C_1 is the R-R 1-form which couples to a D0-brane. This implies that C_1 have Chan-Paton factor σ_3 since T, \bar{T} have Chan-Paton factor $\sigma_1 \pm i\sigma_2$ respectively. At first sight you may think such an idea is not acceptable, but

¹⁶ This is almost the same calculation as that in appendix B of [24]. Thus we omit its detail.

it is required to reproduce the correct spectrum of open strings and satisfy the Cardy's condition as we will see in the next subsection. Therefore we regard our results as another evidence of such an idea. Similar “Chan-Paton factor for closed string vertex” was discussed [7, 1] in the case of a non-BPS D-brane¹⁷, where it was argued that the branch cut due to a RR-vertex provides an extra Chan-Paton factor if its one end is on the non-BPS D-brane. We also argue that some of the states in NSNS sector have the Chan-Paton factor σ_3 . This fact can also be verified by the Cardy's condition and will be required due to the supersymmetry of the bulk theory. Therefore in these sectors we should insert σ_3 in the trace.

Then the tachyon condensation switches not only (3.20) but also

$$W_{\sigma_3}(\alpha, \beta) \propto \sin\left(\frac{\pi\alpha w_\phi^1}{2}\right) \sin\left(\frac{\pi\beta w_\phi^2}{2}\right), \quad (3.22)$$

which is obtained by inserting σ_3 in the trace.

Then by switching both contributions we obtain the following boundary state¹⁸

$$\begin{aligned} |B(\alpha, \beta), + \rangle_{NSNS} &= 2 \exp\left[-\sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} \phi_{-n}^i \tilde{\phi}_n^i\right] \exp\left[+i \sum_{n=1}^{\infty} \sum_{i=1}^2 \eta_{-n+\frac{1}{2}}^i \tilde{\eta}_{-n+\frac{1}{2}}^i\right] \\ &\times \sum_{\vec{w}_\phi \in \mathbf{Z}^2} \left[\cos(\pi\alpha w_\phi^1) \cos(\pi\beta w_\phi^2) \right. \\ &\quad \left. + \sin(\pi\alpha w_\phi^1) \sin(\pi\beta w_\phi^2) \right] |\vec{0}, 2\vec{w}_\phi \rangle_{NSNS}^{(0)}, \end{aligned} \quad (3.23)$$

$$\begin{aligned} |B(\alpha, \beta), + \rangle_{RR} &= 2 \exp\left[-\sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} \phi_{-n}^i \tilde{\phi}_n^i\right] \exp\left[+i \sum_{n=1}^{\infty} \sum_{i=1}^2 \eta_{-n}^i \tilde{\eta}_{-n}^i\right] \\ &\times \sum_{\vec{w}_\phi \in \mathbf{Z}^2} \left[\cos\left\{\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right\} \cos\left\{\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right\} \right. \\ &\quad \left. + \sin\left\{\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right\} \sin\left\{\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right\} \right] |\vec{0}, 2\vec{w}_\phi + \vec{1}, + \rangle_{RR}^{(0)}. \end{aligned} \quad (3.24)$$

This satisfies $|B(0, 0), + \rangle_{NSNS, RR} = |D2 - \overline{D2}, + \rangle_{NSNS, RR}$.

As explained in section 2.1 the point $\alpha = \beta = 1$ is expected to be identified as a $D0 - \overline{D0}$ system [15] where a D0-brane and a $\overline{D0}$ -brane are produced at $(x_1, x_2) = (0, 0)$, (π, π) respectively. The boundary state of this $D0 - \overline{D0}$ system is

$$|D0 - \overline{D0}, + \rangle_{NSNS} = 2 \exp\left[\sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_Y)_{-n}^i (\tilde{\alpha}_Y)_{-n}^i\right] \exp\left[-i \sum_{n=1}^{\infty} \sum_{i=1}^2 \psi_{-n+\frac{1}{2}}^i \tilde{\psi}_{-n+\frac{1}{2}}^i\right]$$

¹⁷For example, in the case of the non-BPS D2-brane the Wess-Zumino term is written as $\int C_1 \wedge dT$ [31].

¹⁸Strictly speaking, from the above explanation we can't decide the relative normalization between the first and the second term. This is determined by the vacuum energy calculation in the next subsection.

$$\times \sum_{\vec{n}_Y \in \mathbf{Z}^2} |\vec{n}_Y, \vec{0}\rangle_{NSNS}^{(0)}, \quad (3.25)$$

$$\begin{aligned} |D0 - \overline{D0}, +\rangle_{RR} &= 2 \exp \left[\sum_{n=1}^{\infty} \sum_{i=1}^2 \frac{1}{n} (\alpha_Y)_n^i (\tilde{\alpha}_Y)_n^i \right] \exp \left[-i \sum_{n=1}^{\infty} \sum_{i=1}^2 \psi_{-n}^i \tilde{\psi}_{-n}^i \right] \\ &\times \sum_{\vec{n}_Y \in \mathbf{Z}^2} |\vec{n}_Y + \frac{\vec{1}}{2}, \vec{0}, +\rangle_{RR}^{(0)}. \end{aligned} \quad (3.26)$$

On the other hand, at this point the zero mode parts of eq.(3.23) and (3.24) become respectively

$$\begin{aligned} \sum_{\vec{w}_\phi \in \mathbf{Z}^2} (-1)^{w_\phi^1 + w_\phi^2} |\vec{0}, 2\vec{w}_\phi\rangle_{NSNS}^{(0)}, \\ \sum_{\vec{w}_\phi \in \mathbf{Z}^2} (-1)^{w_\phi^1 + w_\phi^2} |\vec{0}, 2\vec{w}_\phi + \vec{1}\rangle_{RR}^{(0)}. \end{aligned} \quad (3.27)$$

Then just as we have verified that eq.(3.12) and (3.13) are equivalent to eq. (3.10),(3.11), we can also verify that $|B(1,1), +\rangle_{NSNS,RR}$ is equivalent to eq.(3.25),(3.26) in the same way. For example, eq.(3.25),(3.26) indeed satisfy the following equations which represent the boundary conditions of D0-branes :

$$\partial_1 Y^i(w)|_{\sigma_2=0} |B(1,1), +\rangle_{NSNS,RR} = 0, \quad (3.28)$$

$$\left(\psi_R^i(w) + i\psi_L^i(\bar{w}) \right) |_{\sigma_2=0} |B(1,1), +\rangle_{NSNS,RR} = 0, \quad (i = 1, 2). \quad (3.29)$$

In other words the tachyon condensation from $\alpha = \beta = 0$ to $\alpha = \beta = 1$ changes the boundary conditions (3.14),(3.15) into (3.28),(3.29) and the crucial difference between them is that the latter has the phase factor $(-1)^{w_\phi^1 + w_\phi^2}$. At $\alpha = \beta = 0$ only the first term of eq.(3.24) is nonzero and this corresponds to the RR charge of D2-brane. As the tachyon is condensed the second term also ceases to be zero and this means¹⁹ that the RR charge of the D0-brane is generated. Finally at $\alpha = \beta = 1$ only the second term is nonzero and this is the pure D0-brane RR charge. Note that if we ignored the factor (3.22) which corresponds to σ_3 sector, then the RR-sector boundary state would vanish at $\alpha = \beta = 1$ and be inconsistent. In this way we see explicitly in the closed string formalism that a tachyon kink on a brane-antibrane system produces a codimension two D-brane (see Figure 1).

Let us turn to the other points of α, β . It is easy to see that at $(\alpha, \beta) = (0, 1), (1, 0)$ the RR-sector boundary state does vanish and each system corresponds to a non-BPS D1-brane stretching along the direction of Y^1 or Y^2 respectively (see Figure 1). Physically this can be interpreted as the statement that a tachyon kink produces a codimension one (non-BPS) D-brane. All of these identifications will be verified further by the calculation of vacuum amplitudes including the detailed normalization.

¹⁹It is easy to see that if α, β are small, then the second term is proportional to $V_{T1} V_{T2} |D2\rangle_{RR} \sim |D0\rangle_{RR}$.

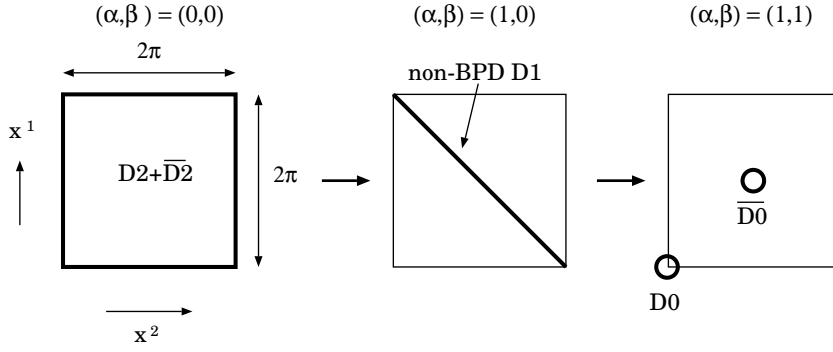


Figure 1: The tachyon condensation in $D2 - \overline{D2}$ system

3.2 Calculation of the vacuum amplitude

Here we calculate the vacuum amplitude of $D2 - \overline{D2}$ system for every value of α, β and translate it from the viewpoint of open string. As a result it will be shown that the boundary state have the correct normalizations or equally correct NSNS and RR-charge needed for the identification and that the additional NSNS and RR sector discussed before are indeed required in order to satisfy the Cardy's condition.

First let us define the propagator for closed string as

$$\Delta = \frac{1}{2} \int_0^\infty ds e^{-sH_c} \sim \frac{1}{k^2} + \dots, \quad (3.30)$$

where H_c denotes the closed string Hamiltonian and its explicit form is given as

$$\begin{aligned} H_c = & \sum_{i=1,2} \left(\frac{(n_\phi^i)^2}{2R_i^2} + \frac{1}{2} (w_\phi^i)^2 R_i^2 \right) + \sum_{i=1,2} \left\{ \sum_n (\phi_{-n}^i \phi_n^i + \tilde{\phi}_{-n}^i \tilde{\phi}_n^i) + \sum_r (\eta_{-r}^i \eta_r^i + \tilde{\eta}_{-r}^i \tilde{\eta}_r^i) \right\} \\ & + \sum_{i=3}^9 \frac{1}{2} (k^i)^2 + \sum_{i=0,3}^7 \left\{ \sum_n (\alpha_{-n}^i \alpha_n^i + \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i) + \sum_r (\chi_{-r}^i \chi_r^i + \tilde{\chi}_{-r}^i \tilde{\chi}_r^i) \right\} + a, \end{aligned} \quad (3.31)$$

where a denotes the zero-energy for each sector and is given as $a = -1$ for NSNS-sector and $a = 0$ for RR-sector.

Then the vacuum amplitudes for NSNS and RR sector are

$$\begin{aligned} Z_{NSNS} &= \frac{(T_{p=2})^2}{16} \int_0^\infty ds \int \left(\frac{dk}{2\pi} \right)^7 \left(\frac{dk'}{2\pi} \right)^7 \{ \langle B(\alpha, \beta), +, k | \Delta | B(\alpha, \beta), +, k' \rangle_{NSNS} \\ &\quad - \langle B(\alpha, \beta), +, k | \Delta | B(\alpha, \beta), -, k' \rangle_{NSNS} \}, \\ &= \frac{(T_{p=2})^2 V_{D2}}{4} \int_0^\infty \frac{1}{(2\pi s)^{\frac{7}{2}}} \left[\sum_{w_\phi^1, w_\phi^2} \{ \cos^2(\pi \alpha w_\phi^1) \cos^2(\pi \beta w_\phi^2) \} \right] \end{aligned}$$

$$\begin{aligned}
& + \sin^2(\pi\alpha w_\phi^1) \sin^2(\pi\beta w_\phi^2) \} q^{(w_\phi^1)^2 + (w_\phi^2)^2} \frac{f_3(q)^8}{f_1(q)^8} - 2 \frac{f_4(q)^6 f_3(q)^2}{f_1(q)^6 f_2(q)^2} \Big], \\
Z_{RR} &= -(T_{p=2})^2 \int_0^\infty ds \int \left(\frac{dk}{2\pi} \right)^7 \left(\frac{dk'}{2\pi} \right)^7 \langle B(\alpha, \beta), +, k | \Delta | B(\alpha, \beta), +, k' \rangle_{RR}, \\
&= -\frac{(T_{p=2})^2 V_{D2}}{4} \int_0^\infty \frac{1}{(2\pi s)^{\frac{7}{2}}} \left[\sum_{w_\phi^1, w_\phi^2} \left\{ \cos^2\left(\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right) \cos^2\left(\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right) \right. \right. \\
&\quad \left. \left. + \sin^2\left(\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right) \sin^2\left(\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right) \right\} q^{(w_\phi^1 + \frac{1}{2})^2 + (w_\phi^2 + \frac{1}{2})^2} \frac{f_2(q)^8}{f_1(q)^8} \right], \quad (3.32)
\end{aligned}$$

where $V_{D2} = (2\pi)^2 V$ is the volume of D2-brane and we defined

$$\begin{aligned}
f_1(q) &= q^{\frac{1}{12}} \prod_{n=1}^\infty (1 - q^{2n}) \quad , \quad f_2(q) = \sqrt{2} q^{\frac{1}{12}} \prod_{n=1}^\infty (1 + q^{2n}), \\
f_3(q) &= q^{-\frac{1}{24}} \prod_{n=1}^\infty (1 + q^{2n-1}) \quad , \quad f_4(q) = q^{-\frac{1}{24}} \prod_{n=1}^\infty (1 - q^{2n-1}), \quad (3.33)
\end{aligned}$$

with $q = e^{-s}$.

Next let us perform modular transformations and interpret this as the open string cylinder amplitude. We define the modulus of the cylinder as $t = \frac{\pi}{s}$ and introduce $\tilde{q} = e^{-\pi t}$. Then we get the following open string amplitude

$$\begin{aligned}
Z_{NS}^{open} &= 2^{-\frac{3}{2}} \pi^{-1} V \int_0^\infty dt \, t^{-\frac{3}{2}} \sum_{n_1, n_2} \left[\frac{1}{2} (\tilde{q}^{n_1^2 + n_2^2} + \tilde{q}^{(n_1 - \alpha)^2 + (n_2 - \beta)^2}) \frac{f_3(\tilde{q})^8}{f_1(\tilde{q})^8} \right. \\
&\quad \left. - \frac{1}{2} (-1)^{n_1 + n_2} (\tilde{q}^{n_1^2 + n_2^2} + \tilde{q}^{(n_1 - \alpha)^2 + (n_2 - \beta)^2}) \frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8} \right], \\
Z_R^{open} &= -2^{-\frac{3}{2}} \pi^{-1} V \int_0^\infty dt \, t^{-\frac{3}{2}} \sum_{n_1, n_2} \tilde{q}^{n_1^2 + n_2^2} \frac{f_2(\tilde{q})^8}{f_1(\tilde{q})^8}, \quad (3.34)
\end{aligned}$$

where we used the following identities

$$\begin{aligned}
\sum_n q^{n^2} &= f_1(q) f_3(q)^2 \quad , \quad \sum_n (-1)^n q^{n^2} = f_1(q) f_4(q)^2, \\
\sum_n q^{(n - \frac{1}{2})^2} &= f_1(q) f_2(q)^2 \quad , \quad f_2(q) f_3(q) f_4(q) = \sqrt{2}, \quad (3.35)
\end{aligned}$$

and the modular properties

$$\begin{aligned}
f_1(e^{-\frac{\pi}{t}}) &= \sqrt{t} f_1(e^{-\pi t}) \quad , \quad f_2(e^{-\frac{\pi}{t}}) = f_4(e^{-\pi t}), \\
f_3(e^{-\frac{\pi}{t}}) &= f_3(e^{-\pi t}) \quad , \quad f_4(e^{-\frac{\pi}{t}}) = f_2(e^{-\pi t}). \quad (3.36)
\end{aligned}$$

Now it is obvious that for each value of α, β the open string spectrum is well defined only if we incorporate the additional sector of the boundary state defined in the previous subsection, otherwise the number of open string states for given n_1, n_2, H_c would be

fractional. This fact will be more clear if we note that this amplitude can be rewritten as

$$\begin{aligned} Z &= Z_{NS}^{open} + Z_R^{open}, \\ &= V_{D2} \int_0^\infty \frac{dt}{t} Tr_{NS-R} \left[\frac{1 + (-1)^{F^\phi} h_1^\phi h_2^\phi}{2} \tilde{q}^{-2H_o} \right], \end{aligned} \quad (3.37)$$

where Tr is the trace over the open string Hilbert space including zero-modes and Chan-Paton sectors. Also H_o means the open string Hamiltonian and is given as follows

$$\begin{aligned} H_o &= (p^0)^2 + \sum_{i=1,2} R_i^2 (w_\phi^i)^2 + \sum_{i=1,2} \left\{ \sum_n \phi_{-n}^i \phi_n^i + \sum_r \eta_{-r}^i \eta_r^i \right\} \\ &\quad + \sum_{i=0,3}^7 \left\{ \sum_n \alpha_{-n}^i \alpha_n^i + \sum_r \chi_{-r}^i \chi_r^i \right\} + a, \end{aligned} \quad (3.38)$$

where a denotes the zero-energy and is given as $a = -\frac{1}{2}$ for NS-sector and $a = 0$ for R-sector.

This physically important constraint (3.37) is generally called Cardy's condition [23]. Also notice that the above open string spectrum is consistent with the momentum shift shown in Table 1.

Finally let us verify the identification at particular α, β . In the case of $(\alpha, \beta) = (1, 0)$ or $(0, 1)$ we get after the modular transformations

$$\begin{aligned} Z_{\alpha=1, \beta=0} = Z_{\alpha=0, \beta=1} &= 2^{-\frac{3}{2}} \pi^{-1} V \int_0^\infty dt t^{-\frac{3}{2}} \sum_{n_1, n_2} \tilde{q}^{n_1^2 + n_2^2} \frac{f_3(\tilde{q})^8 - f_2(\tilde{q})^8}{f_1(\tilde{q})^8}, \\ &= 2\pi\sqrt{2}V \int_0^\infty \frac{dt}{t} Tr_{NS-R}(\tilde{q}^{-2H_o}). \end{aligned} \quad (3.39)$$

Therefore we can identify the system as a non-BPS D1-brane of which length is $2\sqrt{2}\pi$ as expected. Another case is $(\alpha, \beta) = (1, 1)$ and the amplitude can be written as

$$\begin{aligned} Z_{\alpha=1, \beta=1} &= 2^{-\frac{3}{2}} \pi^{-1} V \int_0^\infty dt t^{-\frac{3}{2}} \sum_{n_1, n_2} \left[\tilde{q}^{n_1^2 + n_2^2} \frac{f_3(\tilde{q})^8 - f_2(\tilde{q})^8}{f_1(\tilde{q})^8} - (-1)^{n_1 + n_2} \tilde{q}^{n_1^2 + n_2^2} \frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8} \right], \\ &= 2V \int_0^\infty \frac{dt}{t} \frac{1}{\sqrt{8\pi^2 t}} \sum_{m_1, m_2} \left[\tilde{q}^{2m_1^2 + 2m_2^2} \frac{f_3(\tilde{q})^8 - f_4(\tilde{q})^8 - f_2(\tilde{q})^8}{2f_1(\tilde{q})^8} \right. \\ &\quad \left. + \tilde{q}^{2(m_1 + \frac{1}{2})^2 + 2(m_2 + \frac{1}{2})^2} \frac{f_3(\tilde{q})^8 + f_4(\tilde{q})^8 - f_2(\tilde{q})^8}{2f_1(\tilde{q})^8} \right], \\ &= 2V \int_0^\infty \frac{dt}{t} Tr_{NS-R} \left[\frac{1 + (-1)^F}{2} \tilde{q}^{-2H_c} \right] + 2V \int_0^\infty \frac{dt}{t} Tr_{NS-R} \left[\frac{1 - (-1)^F}{2} \tilde{q}^{-2H_c} \right], \end{aligned} \quad (3.40)$$

where we have defined $m_1 = \frac{n_1 + n_2}{2}$, $m_2 = \frac{n_1 - n_2}{2}$. This shows explicitly that the system is equivalent to a D0-brane and an anti D0-brane which are separated from each other by $\Delta x_1 = \Delta x_2 = \pi$.

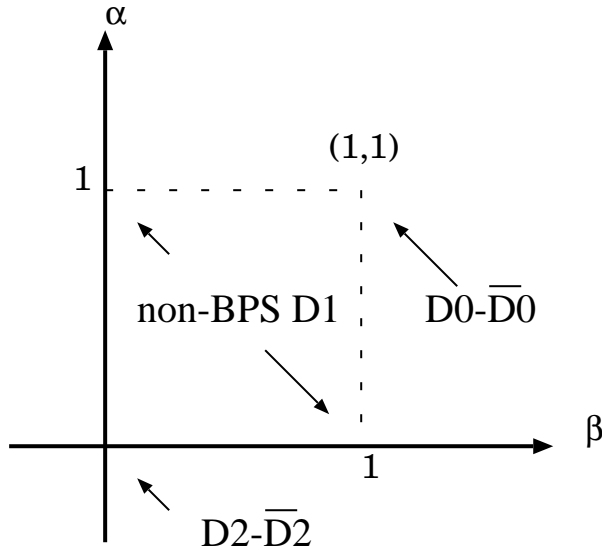


Figure 2: The moduli space of the non-supersymmetric D-brane configurations

3.3 Moduli space of non-supersymmetric D-branes

So far we have discussed the boundary states which describe various tachyon condensations in $D2 - \overline{D2}$ system at the critical radii. The tachyon condensations are parameterized by α, β which have periodicity $\alpha \sim \alpha + 2, \beta \sim \beta + 2$. At this particular radius the non-supersymmetric D-brane configurations for all values of α, β are realized in conformal invariant manners. Figure 2 shows the moduli space of the non-supersymmetric D-brane configurations. In particular $(\alpha, \beta) = (0, 0) \rightarrow (1, 0) \rightarrow (1, 1)$ can be regarded as a continuous version of the descent relation [9] (see Figure 1).

Realistically we are interested in the $D2 - \overline{D2}$ system at generic radii. If we shift the radius, tadpoles develop as can be seen by the method discussed in [5, 9, 15] or by computing one point functions using the boundary state we constructed. The tadpoles only vanish at $\sin(\pi\alpha) = \sin(\pi\beta) = 0$, which correspond to $D2 - \overline{D2}$, non-BPS $D1$ -brane and $D0 - \overline{D0}$.

3.4 Tachyon condensation in general $Dp - \overline{Dp}$ systems

In this subsection, we generalize our construction to the higher codimension cases. It is enough to consider the $D8 - \overline{D8}$ system (In odd codimension case, we have only to consider the decay to the $D(-1) - \overline{D(-1)}$ system.). The different points from the analysis of $D2 - \overline{D2}$ system are the slightly complicated choice of the gamma matrices

and Chan-Paton factors. We mainly concentrate on these issues in the presentation here.

In particular, we deal with the case where 2^3 $D8 - \overline{D8}$ pairs at critical radii ($R_1 = \dots = R_8 = 1$) become 2^3 $D0 - \overline{D0}$ pairs via the tachyon condensation as the marginal deformation. These 2^3 soliton - antisoliton pairs represent 2^3 $D0 - \overline{D0}$ pairs, and a single $D0$ brane is identified with the single codimension 8 soliton on the $D8 - \overline{D8}$ pair at the point where the tachyon condensation is maximum. Many other points are identified with the bound states of $D8 - \overline{D8}$, $D6 - \overline{D6}$, $D4 - \overline{D4}$, $D2 - \overline{D2}$, $D0 - \overline{D0}$ at critical radii. We show the emergence of the RR charges of the lower dimensional D -branes explicitly.

We switched on \mathbf{Z}_2 Wilson line to the second anti-brane using the Wilson lines [15]

$$\begin{aligned} X^1 X^2 \text{ direction} & \quad \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1, & X^3 X^4 \text{ direction} & \quad \sigma_3 \otimes 1 \otimes 1 \otimes 1, \\ X^5 X^6 \text{ direction} & \quad 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1, & X^7 X^8 \text{ direction} & \quad 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3. \end{aligned} \quad (3.41)$$

where we adopt the following representation of the $SO(8)$ Clifford algebra

$$\begin{aligned} \Gamma_1 &= 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_2, & \Gamma_2 &= \sigma_3 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2, & \Gamma_3 &= \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2, \\ \Gamma_4 &= \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2, & \Gamma_5 &= 1 \otimes \sigma_3 \otimes \sigma_2 \otimes \sigma_2, & \Gamma_6 &= 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2, \\ \Gamma_7 &= 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_2, & \Gamma_8 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1. \end{aligned} \quad (3.42)$$

Above matrices for the Wilson lines have the properties such that the matrix in the $X^{2k+1} X^{2k+2}$ direction anticommutes with $\Gamma_{2k+1}, \Gamma_{2k+2}$, and commutes with the other six gamma matrices. Since we have seen that the oscillator parts don't contribute crucially the tachyon condensation, we omit these parts. The zero mode part of the boundary states of this $D8 - \overline{D8}$ system is given by

$$|D8 - \overline{D8}, \gamma\rangle_{NSNS}^{(0)} = \sum_{\vec{w}_X \in \mathbf{Z}^8} \left(1 + (-1)^{w_X^1 + w_X^2}\right) \left(1 + (-1)^{w_X^3 + w_X^4}\right) \quad (3.43)$$

$$\begin{aligned} & \times \left(1 + (-1)^{w_X^5 + w_X^6}\right) \left(1 + (-1)^{w_X^7 + w_X^8}\right) |\vec{0}, \vec{w}_X\rangle_{NSNS}^{(0)}, \\ |D8 - \overline{D8}, \gamma\rangle_{RR}^{(0)} &= \sum_{\vec{w}_X \in \mathbf{Z}^8} \left(1 - (-1)^{w_X^1 + w_X^2}\right) \left(1 - (-1)^{w_X^3 + w_X^4}\right) \quad (3.44) \\ & \times \left(1 - (-1)^{w_X^5 + w_X^6}\right) \left(1 - (-1)^{w_X^7 + w_X^8}\right) |\vec{0}, \vec{w}_X, \gamma\rangle_{RR}^{(0)}. \end{aligned}$$

Now we change the variables as follows

$$\begin{aligned} Y^{2k+1} &= \frac{1}{\sqrt{2}} (X^{2k+1} + X^{2k+2}), & Y^{2k+2} &= \frac{1}{\sqrt{2}} (X^{2k+1} - X^{2k+2}), \\ \psi_{R,L}^{2k+1} &= \frac{1}{\sqrt{2}} (\chi_{R,L}^{2k+1} + \chi_{R,L}^{2k+2}), & \psi_{R,L}^{2k+2} &= \frac{1}{\sqrt{2}} (\chi_{R,L}^{2k+1} - \chi_{R,L}^{2k+2}), \end{aligned}$$

where $k = 0, 1, 2, 3$. In terms of these variables, the zero mode parts of the boundary state for $D8 - \overline{D8}$ system are rewritten as follows

$$|D8 - \overline{D8}, \gamma\rangle_{NSNS}^{(0)} = 16 \sum_{\vec{w}_Y \in \mathbf{Z}^8} |\vec{0}, 2\vec{w}_Y\rangle_{NSNS}^{(0)}, \quad (3.45)$$

$$|D8 - \overline{D8}, \gamma\rangle_{RR}^{(0)} = 16 \sum_{\vec{w}_Y \in \mathbf{Z}^8} |\vec{0}, 2\vec{w}_Y + \vec{1}, \gamma\rangle_{RR}^{(0)}. \quad (3.46)$$

where $w_Y^{2k+1} = w_X^{2k+1} + w_X^{2k+2}$, $w_Y^{2k+2} = w_X^{2k+1} - w_X^{2k+2}$.

In order to describe the effect of the tachyon condensation, we change the variables using bosonization techniques. First, by fermionization Y^i are represented by fermions ξ^i, η^i and next, by bosonization we introduce free bosons ϕ^i . In the following we neglect the cocycle factors, but we can take their roles into account giving gamma matrix $\tilde{\Gamma}_i$ to $e^{i\sqrt{2}Y^i}$, and $\tilde{\Gamma}_{12345678}$ to ψ^i, η^i etc. Then the $D8 - \overline{D8}$ system at critical radii is described with the following projection as in section 2.1

$$(-1)^F h_1 \dots h_8 = (-1)^{F\phi} h_1^\phi \dots h_8^\phi = 1. \quad (3.47)$$

Using (ϕ, η) modes, we can write down the boundary states of $D8 - \overline{D8}$ system for $\gamma = +1$ and their zero modes are given by

$$|D8 - \overline{D8}, +\rangle_{NSNS}^{(0)} = 16 \sum_{\vec{w}_\phi \in \mathbf{Z}^8} |\vec{0}, 2\vec{w}_\phi\rangle_{NSNS}^{(0)}, \quad (3.48)$$

$$|D8 - \overline{D8}, +\rangle_{RR}^{(0)} = 16 \sum_{\vec{w}_\phi \in \mathbf{Z}^8} |\vec{0}, 2\vec{w}_\phi + \vec{1}, +\rangle_{RR}^{(0)}. \quad (3.49)$$

The equivalence of two states written with different variables is provided by the fact that these states satisfy the definition equation of the boundary state for $D8 - \overline{D8}$ system. Also, the boundary states for $\gamma = -1$ is given as in the previous subsection.

Now we are ready to condense the tachyon. The tachyon condensation is represented as the insertion of the following Wilson line [15]

$$\exp \left[\sum_{i=1}^8 \frac{i\alpha_i}{2\sqrt{2}} \oint \partial\phi^i \otimes \Gamma_i \right], \quad (3.50)$$

where α_i represent the parameters of the condensation and have the periodicity $\alpha_i \sim \alpha_i + 2$. This represents the marginal deformation at the critical radius. These traces with the insertion of the various Chan-Paton factors are given as the following five types

$$\begin{aligned} W_1(\{\alpha_i\}) &: \prod_{i=1}^8 \cos \left(\frac{\pi\alpha_i w_\phi^i}{2} \right), \\ W_{\Gamma_{ij}}(\{\alpha_i\}) &: \left[\prod_{i=1}^6 \cos \left(\frac{\pi\alpha_i w_\phi^i}{2} \right) \right] \sin \left(\frac{\pi\alpha_7 w_\phi^7}{2} \right) \sin \left(\frac{\pi\alpha_8 w_\phi^8}{2} \right), \quad {}_8C_2 = 28 \text{ combinations}, \\ W_{\Gamma_{ijkl}}(\{\alpha_i\}) &: \left[\prod_{i=1}^4 \cos \left(\frac{\pi\alpha_i w_\phi^i}{2} \right) \right] \left[\prod_{j=5}^8 \sin \left(\frac{\pi\alpha_j w_\phi^j}{2} \right) \right], \quad {}_8C_4 = 70 \text{ combinations}, \\ W_{\Gamma_{ijklmn}}(\{\alpha_i\}) &: \cos \left(\frac{\pi\alpha_1 w_\phi^1}{2} \right) \cos \left(\frac{\pi\alpha_2 w_\phi^2}{2} \right) \left[\prod_{i=3}^8 \sin \left(\frac{\pi\alpha_i w_\phi^i}{2} \right) \right], \quad {}_8C_2 = 28 \text{ combinations}, \\ W_{\Gamma_{12345678}}(\{\alpha_i\}) &: \prod_{i=1}^8 \sin \left(\frac{\pi\alpha_i w_\phi^i}{2} \right). \end{aligned} \quad (3.51)$$

Again the insertion these operators in front of the boundary state corresponds to the tachyon condensation in the closed string sector.

This can be understood from the following Wess-Zumino coupling [30]

$$\int_{D8-\overline{D8}} C_{RR} \wedge \text{STr } e^{\mathcal{F}}, \quad \mathcal{F} = \begin{pmatrix} F^+ - T\bar{T} & DT \\ D\bar{T} & F^- - \bar{T}T \end{pmatrix} \quad (3.52)$$

where $D = d + A^+ - A^-$ and T, \bar{T} represent the complex tachyon field. The fields A^+, A^- denote the gauge fields on the brane, anti-brane respectively, which is 0 in this case. Combining with the fact that the tachyon configuration is given by [17, 15]

$$\text{for example} \quad T(x) \sim \Gamma_i x_i, \quad \text{at } x_i \sim 0, \quad (3.53)$$

we can speculate that the RR fields C_{2k+1} should have the following Chan-Paton factors

$$C_7 : \Gamma_{ij}, \quad C_5 : \Gamma_{ijkl}, \quad C_3 : \Gamma_{ijklmn}, \quad C_1 : \Gamma_{12345678}. \quad (3.54)$$

For example, this can be understood as the following expression

$$\int_{D8-\overline{D8}} C_{D0} dx^1 \wedge dx^2 \wedge \dots \wedge dx^8 \Gamma_{12345678}. \quad (3.55)$$

Thus the traces (3.51) correspond to the closed string sector that belongs to the each Chan-Paton factor. Thus the generation of the lower dimensional D -branes' charges is induced by the above Wess-Zumino terms which are characteristic of the brane-antibrane systems.

Switching on the above operators, we obtain the following boundary states

$$|B(\{\alpha_i\}), +\rangle_{NSNS}^{(0)} = 16 \sum_{\vec{w}_\phi \in \mathbf{Z}^8} \left[K(\{m_i\} = \{w_\phi^i\}) \right] |\vec{0}, 2\vec{w}_\phi\rangle_{NSNS}^{(0)}, \quad (3.56)$$

$$|B(\{\alpha_i\}), +\rangle_{RR}^{(0)} = 16 \sum_{\vec{w}_\phi \in \mathbf{Z}^8} \left[K(\{m_i\} = \{w_\phi^i + \frac{1}{2}\}) \right] |\vec{0}, 2\vec{w}_\phi + \vec{1}, +\rangle_{RR}^{(0)}, \quad (3.57)$$

where

$$\begin{aligned} K(\{m_i\}) &= \prod_{i=1}^8 \cos(\pi \alpha_i m_i) + \left[\prod_{i=1}^6 \cos(\pi \alpha_i m_i) \cdot \sin(\pi \alpha_7 m_7) \sin(\pi \alpha_8 m_8) + 27 \text{ terms} \right] \\ &+ \left[\prod_{i=1}^4 \cos(\pi \alpha_i m_i) \cdot \prod_{j=5}^8 \sin(\pi \alpha_j m_j) + 69 \text{ terms} \right] \\ &+ \left[\cos(\pi \alpha_1 m_1) \cos(\pi \alpha_2 m_2) \cdot \prod_{i=3}^8 \sin(\pi \alpha_i m_i) + 27 \text{ terms} \right] + \prod_{i=1}^8 \sin(\pi \alpha_i m_i). \end{aligned} \quad (3.58)$$

We have $|B(\{0\}), +\rangle_{NSNS,RR} = |D8 - \overline{D8}, +\rangle_{NSNS,RR}$.

At $\alpha_i = 1$ for all the i , the zero mode parts of the above boundary states become

$$\text{NSNS sector} \quad \sum_{\vec{w}_\phi \in \mathbf{Z}^8} (-1)^{w_\phi^1 + \dots + w_\phi^8} |\vec{0}, 2\vec{w}_\phi\rangle_{NSNS}^{(0)}, \quad (3.59)$$

$$\text{RR sector} \quad \sum_{\vec{w}_\phi \in \mathbf{Z}^8} (-1)^{w_\phi^1 + \dots + w_\phi^8} |\vec{0}, 2\vec{w}_\phi + \vec{1}, +\rangle_{RR}^{(0)}. \quad (3.60)$$

The boundary state of this system is rewritten as follows

$$|D0 - \overline{D0}, +\rangle_{NSNS}^{(0)} = 16 \sum_{\vec{n}_Y \in \mathbf{Z}^8} |\vec{n}_Y, \vec{0}\rangle_{NSNS}^{(0)}, \quad (3.61)$$

$$|D0 - \overline{D0}, +\rangle_{RR}^{(0)} = 16 \sum_{\vec{n}_Y \in \mathbf{Z}^8} |\vec{n}_Y + 1/2, \vec{0}, +\rangle_{RR}^{(0)}. \quad (3.62)$$

These boundary states satisfy the definition equation (the boundary conditions) for the $D0 - \overline{D0}$ system.

Again, the extra phase factor $(-1)^{w_\phi^1 + \dots + w_\phi^8}$ changes the boundary condition from $D8 - \overline{D8}$ to $D0 - \overline{D0}$. This corresponds to the fact that $D0$ -branes and $\overline{D0}$ -branes are produced at each choices of

$$(x_{2k+1}, x_{2k+2}) = (0, 0) \quad \text{or} \quad (\pi, \pi). \quad (3.63)$$

Around the above points, there exists a soliton (anti-soliton) if the number of coordinate pairs taking the value (π, π) is even (odd). Finally evaluating the vacuum amplitudes for NSNS and RR sector with all the ghosts taking into account, it is easy to check the Cardy's constraint explicitly. Thus we have established in the closed string viewpoint that a tachyonic soliton on the $D8 - \overline{D8}$ system produces a codimension eight $D0$ -branes.

Next turn to the other points of α_i . The tadpole cancellation restricts the admissible values to $\alpha_i = 0, 1$. We note the following basic observation.

	$\alpha = 0$	$\alpha = 1$
$\cos(\pi\alpha w)$	1	$(-1)^w$
$\sin(\pi\alpha w)$	0	0
$\cos\left(\pi\alpha\left(w + \frac{1}{2}\right)\right)$	1	0
$\sin\left(\pi\alpha\left(w + \frac{1}{2}\right)\right)$	0	$(-1)^w$

(3.64)

Then in the case when $2n$ of α_i is equal to 1, the system corresponds to the $D(8 - 2n) - \overline{D(8 - 2n)}$ system. The number of such configurations corresponds to the possible choice of the Chan-Paton factor in the closed string sector. For example, 28 Γ_{ij} corresponds to the degree of freedom in order to set the direction of the codimension 2 among 8 directions. On the other hand, when the odd number of α_i is equal to 1, the boundary states in RR sector vanish and the system corresponds to a non-BPS D -branes.

Again we emphasize that the Chan-Paton factors in the closed string sector played the crucial role in our analysis.

4 Boundary state description of tachyon condensation in T^4/\mathbf{Z}_2 orbifold theory

In this section we construct the boundary state description of tachyon condensations in the orbifold theory. We first discuss the decay mode from a $D0 - \overline{D0}$ system to a non-BPS D1-brane in detail. Next we extend this result to the higher codimension cases. We also discuss the occurrence of the bose-fermi degeneracy [26, 27] in this formalism.

4.1 Construction of the boundary state

The boundary state which represents $D0 - \overline{D0}$ in T^4/\mathbf{Z}_2 orbifold at the radii $R_6 = \sqrt{2}, R_7 = R_8 = R_9 = R$ is given as follows

$$|B\rangle = \frac{T_{p=0}}{2\sqrt{2}}(|U\rangle_{NSNS} + |U\rangle_{RR}) + \frac{N}{2\sqrt{2}}(|T\rangle_{NSNS} + |T\rangle_{RR}), \quad (4.1)$$

where $|U\rangle, |T\rangle$ denote the untwisted, twisted part of the boundary state. The normalization for twisted sector is determined by comparing the closed string vacuum amplitude with the open string one. The result is given by $N = 2^3\pi^{\frac{3}{2}}$.

The more detailed structure of each sector is written as

$$\begin{aligned} |U\rangle_{NSNS} &= \frac{1}{2} \int \left(\frac{dk}{2\pi}\right)^5 \sum_{n_6, n_7, n_8, n_9} \frac{1 + (-1)^{n_6}}{(2\pi\sqrt{2})(2\pi R)^3} [|U, +, k^i, n\rangle_{NSNS} - |U, -, k^i, n\rangle_{NSNS}], \\ |U\rangle_{RR} &= 2 \int \left(\frac{dk}{2\pi}\right)^5 \sum_{n_6, n_7, n_8, n_9} \frac{1 - (-1)^{n_6}}{(2\pi\sqrt{2})(2\pi R)^3} [|U, +, k^i, n\rangle_{RR} + |U, -, k^i, n\rangle_{RR}], \\ |T\rangle_{NSNS} &= \int \left(\frac{dk}{2\pi}\right)^5 [|T_1, +, k^i\rangle_{NSNS} + |T_1, -, k^i\rangle_{NSNS} + |T_2, +, k^i\rangle_{NSNS} + |T_2, -, k^i\rangle_{NSNS}], \\ |T\rangle_{RR} &= \int \left(\frac{dk}{2\pi}\right)^5 [|T_1, +, k^i\rangle_{RR} + |T_1, -, k^i\rangle_{RR} + |T_2, +, k^i\rangle_{RR} + |T_2, -, k^i\rangle_{RR}], \end{aligned} \quad (4.2)$$

where we set $1 \leq i \leq 5$ and $|T_1\rangle, |T_2\rangle$ represent the twisted sector boundary states corresponding to two different fixed points. As we will explain briefly in the appendix A, $|U, \gamma, k^i, n\rangle$ and $|T_1, +, k^i\rangle, |T_2, +, k^i\rangle$ are defined by the conditions (A.4) expanding the fields (Y, ψ) in each Hilbert space.

Next we need to rewrite the above boundary state in terms of (ϕ, η) in order to describe the tachyon condensation as discussed in the previous section. For the untwisted sector the procedure is almost the same and the result are as follows (we show below only the relevant modes which correspond to x^6 direction and omit the superscript 6 in this

subsection)

$$\begin{aligned}
|U, +\rangle_{NSNS} &= \exp\left(\sum_{n \in \mathbf{Z}} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} - i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \psi_{-r} \tilde{\psi}_{-r}\right) 2 \sum_{n_Y} |2n_Y, 0\rangle_{NSNS}^{(0)} \\
&= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \eta_{-r} \tilde{\eta}_{-r}\right) 2 \sum_{w_\phi} (-1)^{w_\phi} |0, w_\phi\rangle_{NSNS}^{(0)}, \\
|U, +\rangle_{RR} &= \exp\left(\sum_{n \in \mathbf{Z}} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} - i \sum_{r \in \mathbf{Z}} \psi_{-r} \tilde{\psi}_{-r}\right) 2 \sum_{n_Y} |2n_Y + 1, 0, +\rangle_{RR}^{(0)} \\
&= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z}} \eta_{-r} \tilde{\eta}_{-r}\right) 2 \sum_{w_\phi} (-1)^{w_\phi} |0, w_\phi + \frac{1}{2}, +\rangle_{RR}^{(0)},
\end{aligned} \tag{4.3}$$

Next let us turn to the twisted sector. The twist operator σ which map the untwisted sector into twisted sector is needed [32]. A candidate for such an operator is given as

$$\sigma = e^{i \frac{1}{\sqrt{2}} (\phi_R - \tilde{\phi}_L)}, \tag{4.4}$$

which has the desired singular property as

$$\psi(z) \sigma(0) \sim O(z^{-\frac{1}{2}}), \quad \partial X(z) \sigma(0) \sim O(z^{-\frac{1}{2}}), \tag{4.5}$$

This operator leads to the correct boundary condition of twisted sector boundary state. Then we can rewrite the twisted sector boundary state as

$$\begin{aligned}
|T, +\rangle_{NSNS} &= \exp\left(\sum_{n \in \mathbf{Z}} \frac{1}{n + \frac{1}{2}} \alpha_{-(n + \frac{1}{2})} \tilde{\alpha}_{-(n + \frac{1}{2})} - i \sum_{r \in \mathbf{Z}} \psi_{-r} \tilde{\psi}_{-r}\right) \times \{|T_1\rangle_{NSNS}^{(0)} + |T_2\rangle_{NSNS}^{(0)}\} \\
&= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \eta_{-r} \tilde{\eta}_{-r}\right) \sum_{w_\phi} (-1)^{w_\phi} |w_\phi + \frac{1}{2}\rangle_{NSNS}^{(0)}, \\
|T, +\rangle_{RR} &= \exp\left(\sum_{n \in \mathbf{Z}} \frac{1}{n + \frac{1}{2}} \alpha_{-(n + \frac{1}{2})} \tilde{\alpha}_{-(n + \frac{1}{2})} - i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \psi_{-r} \tilde{\psi}_{-r}\right) \times \{|T_1\rangle_{RR}^{(0)} + |T_2\rangle_{RR}^{(0)}\} \\
&= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z}} \eta_{-r} \tilde{\eta}_{-r}\right) \sum_{w_\phi} (-1)^{w_\phi} |w_\phi\rangle_{RR}^{(0)}.
\end{aligned} \tag{4.6}$$

Notice that this transformation or “bosonization” procedure can be verified by showing the bosonized boundary state does indeed satisfy the boundary condition of the original one as in section 3. It is also easy to see that the vacuum amplitude doesn’t change by the bosonization using the relations (3.35).

4.2 Tachyon condensation

Since we have constructed the boundary state of $D0 - \overline{D0}$ system in terms of (ϕ, η) , it is straightforward to determine the boundary state which describe the tachyon condensation process in that system. The Wilson line corresponding to the tachyon condensation discussed in section 2 can be written as

$$W = \text{Tr} \exp\left(\frac{i}{2\sqrt{2}}\alpha \oint \partial\phi \otimes \sigma_1\right) = \cos(\pi w_\phi \alpha). \quad (4.7)$$

Then the effect of the tachyon condensation appears at the coefficients in front of the zero mode parts as in section 3. If we consider the point $\alpha = 1$, which corresponds to the maximal condensation, then the untwisted RR-sector and the twisted NSNS-sector vanish. The untwisted NSNS-sector and the twisted RR-sector become as follows

$$\begin{aligned} |U, +\rangle_{NSNS} &= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \eta_{-r} \tilde{\eta}_{-r}\right) 2 \sum_{w_\phi} |w_\phi\rangle_{NSNS}^{(0)} \\ &= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} + i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \psi_{-r} \tilde{\psi}_{-r}\right) 2 \sum_{w_Y} |w_Y\rangle_{NSNS}^{(0)}, \\ |T, +\rangle_{RR} &= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n} \phi_{-n} \tilde{\phi}_{-n} + i \sum_{r \in \mathbf{Z}} \eta_{-r} \tilde{\eta}_{-r}\right) \sum_{w_\phi} |w_\phi\rangle_{RR}^{(0)} \\ &= \exp\left(\sum_{n \in \mathbf{Z}} -\frac{1}{n + \frac{1}{2}} \alpha_{-(n+\frac{1}{2})} \tilde{\alpha}_{-(n+\frac{1}{2})} + i \sum_{r \in \mathbf{Z} + \frac{1}{2}} \psi_{-r} \tilde{\psi}_{-r}\right) \times \{|T_1\rangle_{RR}^{(0)} + |T_2\rangle_{RR}^{(0)}\}, \end{aligned} \quad (4.8)$$

where we have “rebosonized” the expression using the basis (Y, ψ) . Note that the boundary condition is changed into that of D1-brane because of the extra phase $\cos(\pi w_\phi) = (-1)^{w_\phi}$.

Now it is obvious²⁰ that the above boundary state is the same as that of a non-BPS D1-brane [8] stretching between the fixed points.

In this way the tachyon condensation process from $D0 - \overline{D0}$ to a non-BPS D1-brane (and also its reverse if we replace α with $1 - \alpha$) is explicitly shown by using boundary state formalism. It would be an interesting fact that the twisted sector of (Y, ψ) can be expressed by using the untwisted sector of another field basis (ϕ, η) and this is crucial in the above discussion of the tachyon condensation in the orbifold theory. This fact will also become important if we consider tachyon condensation processes in other orbifold theories.

²⁰If we start a $D0 - \overline{D0}$ which has the different relative twisted charge, then we can show by using almost the same procedure that the final object is a non-BPS D1-brane with a \mathbf{Z}_2 Wilson line after the tachyon condensation.

4.3 Generalization to the higher codimension case

Then we will be interested in the higher codimension cases. As we will show below, such generalizations are not so difficult in our boundary state formalism and the results remain almost the same as in section 3. Therefore the discussion is short.

To make things clear we consider the tachyon condensation that changes two $D4 - \overline{D4}$ pairs into two $D0 - \overline{D0}$ pairs (codimension four). Here $D4 - \overline{D4}$ system has appropriate \mathbf{Z}_2 Wilson lines in the same sense of section 3. This process includes the decay modes into two $D2 - \overline{D2}$ pairs. First let us define the coordinates of T^4 as (x^6, x^7, x^8, x^9) and their $\frac{\pi}{4}$ rotated coordinates as (y^6, y^7, y^8, y^9) . We also take the radii of T^4 as $R^6 = R^7 = R^8 = R^9 = 1$ in terms of the coordinates (x^6, x^7, x^8, x^9) as in the previous discussion in flat space. It is important to note that this $D4 - \overline{D4}$ system can be described in terms of (y^6, y^7, y^8, y^9) as a $D4 - \overline{D4}$ system on T^4/\mathbf{Z}_2 of which radii are all $\sqrt{2}$ with the projection $(-1)^F h_6 h_7 h_8 h_9 = 1$ as in the case of flat space. At this radius we can change the basis (Y^i, ψ^i) into (ϕ^i, η^i) by the bosonization procedures, which are trivial generalizations of eq.(4.3) and (4.6). Then we can describe the tachyon condensation processes and let us denote the corresponding four parameters as $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. Notice that in order to get four parameters²¹ corresponding to the marginal deformation in the four directions we should start with not one but two pieces of $D4 - \overline{D4}$. At the point $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$ both the untwisted and the twisted boundary states gain the same extra phase factor $(-1)^{w_6+w_7+w_8+w_9}$ and the boundary conditions along (y^6, y^7, y^8, y^9) are reversed. Then we get two $D0$ -branes and two anti $D0$ -branes which sit at $(x^6, x^7, x^8, x^9) = (0, 0, 0, 0), (\pi, \pi, \pi, \pi)$ and $(\pi, \pi, 0, 0), (0, 0, \pi, \pi)$ respectively. In this way we find that the tachyon condensation of brane-antibrane system in T^4/\mathbf{Z}_2 orbifold can be treated almost in the same way as in flat space except the treatment of the twisted sector.

4.4 Comments on bose-fermi degeneracy

Finally let us discuss the relation between the boundary state description in this section and the bose-fermi degeneracy [26]. First we compute the vacuum amplitude of the system (4.3),(4.6) with the insertion of the Wilson line (4.7) using (ϕ, η) field representation for x^6 direction. The result is

$$\begin{aligned} & \langle B(\alpha) | \Delta | B(\alpha) \rangle \\ &= \frac{V}{16} \int ds \left(\frac{1}{2\pi s} \right)^{\frac{5}{2}} \left[\frac{\pi^3 2^{\frac{5}{2}}}{R^3} \left\{ \sum_{w_\phi, \vec{n}_Y} q^{w_\phi^2 + \frac{n_Y^2}{2R^2}} \cos^2(\pi \alpha w_\phi) \frac{f_3(q)^8}{f_1(q)^8} - \sum_{\vec{n}_Y} q^{\frac{n_Y^2}{2R^2}} \frac{\sqrt{2} f_3(q) f_4(q)^7}{f_2(q) f_1(q)^7} \right\} \right] \end{aligned}$$

²¹If we started with one $D4 - \overline{D4}$, then we would only get the decay modes into a $D2 - \overline{D2}$ and the codimension four configuration is impossible.

$$\begin{aligned}
& - \sum_{w_\phi, \vec{n}_Y} \cos^2\left(\pi\alpha\left(w_\phi + \frac{1}{2}\right)\right) q^{(w_\phi + \frac{1}{2})^2 + \frac{n_Y^2}{2R^2}} \frac{f_2(q)^8}{f_1(q)^8} \Big\} \\
& + 2^6 \pi^3 \Big\{ \sum_{w_\phi} \cos^2\left(\pi\left(w_\phi + \frac{1}{2}\right)\alpha\right) q^{(w_\phi + \frac{1}{2})^2} \frac{f_3(q)^5 f_2(q)^3}{\sqrt{2} f_1(q)^5 f_4(q)^3} - \sum_{w_\phi} \cos^2(\pi w_\phi \alpha) q^{w_\phi^2} \frac{f_2(q)^5 f_3(q)^3}{\sqrt{2} f_1(q)^5 f_4(q)^3} \Big\},
\end{aligned} \tag{4.9}$$

where $\vec{n}_Y = (n_Y^7, n_Y^8, n_Y^9) \in \mathbf{Z}^3$ are momenta in the directions of (y^7, y^8, y^9) . We can see that this amplitude does vanish if $R = \frac{1}{\sqrt{2}}, \alpha = 1$ and this phenomenon of non-BPS D1-brane is called bose-fermi degeneracy [26]. Below we would like to discuss this from the viewpoint of the boundary state.

The particular radii of torus $R = \frac{1}{\sqrt{2}}$ enable us to perform further bosonization procedures in the direction of x^7, x^8, x^9 . The result is as follows (we only show the zero modes and oscillators which correspond to x^6, x^7, x^8, x^9)

$$\begin{aligned}
|U, +\rangle_{NSNS} &= \exp \left[\sum_{n \in \mathbf{Z}} \frac{1}{n} (-\alpha_{-n}^6 \tilde{\alpha}_{-n}^6 + \sum_{i=7,8,9} \alpha_{-n}^i \tilde{\alpha}_{-n}^i) \right. \\
&\quad \left. - i \sum_{r \in \mathbf{Z} + \frac{1}{2}} (-\psi_{-r}^6 \tilde{\psi}_{-r}^6 + \sum_{i=7,8,9} \psi_{-r}^i \tilde{\psi}_{-r}^i) \right] 2 \sum_{w_\phi^6, \vec{n}_Y} |w_Y^6, n_Y^7, n_Y^8, n_Y^9\rangle_{NSNS}^{(0)}, \\
&= \exp \left[\sum_{n \in \mathbf{Z}} \frac{1}{n} (-\phi_{-n}^6 \tilde{\phi}_{-n}^6 - \sum_{i=7,8,9} \phi_{-n}^i \tilde{\phi}_{-n}^i) \right. \\
&\quad \left. - i \sum_{r \in \mathbf{Z} + \frac{1}{2}} (-\eta_{-r}^6 \tilde{\eta}_{-r}^6 - \sum_{i=7,8,9} \eta_{-r}^i \tilde{\eta}_{-r}^i) \right] 2 \sum_{w_\phi} (-1)^{w_\phi^7 + w_\phi^8 + w_\phi^9} |w_\phi^6, 2w_\phi^7, 2w_\phi^8, 2w_\phi^9\rangle_{NSNS}^{(0)}, \\
|T, +\rangle_{RR} &= \exp \left[\sum_{n \in \mathbf{Z}} \frac{1}{n + \frac{1}{2}} (-\alpha_{-n-\frac{1}{2}}^6 \tilde{\alpha}_{-n-\frac{1}{2}}^6 + \sum_{i=7,8,9} \alpha_{-n-\frac{1}{2}}^i \tilde{\alpha}_{-n-\frac{1}{2}}^i) \right. \\
&\quad \left. - i \sum_{r \in \mathbf{Z} + \frac{1}{2}} (-\psi_{-r}^6 \tilde{\psi}_{-r}^6 + \sum_{i=7,8,9} \psi_{-r}^i \tilde{\psi}_{-r}^i) \right] \{|T_1\rangle_{RR}^{(0)} + |T_2\rangle_{RR}^{(0)}\}, \\
&= \exp \left[\sum_{n \in \mathbf{Z}} \frac{1}{n} (-\phi_{-n}^6 \tilde{\phi}_{-n}^6 - \sum_{i=7,8,9} \phi_{-n}^i \tilde{\phi}_{-n}^i) \right. \\
&\quad \left. - i \sum_{r \in \mathbf{Z}} (-\eta_{-r}^6 \tilde{\eta}_{-r}^6 - \sum_{i=7,8,9} \eta_{-r}^i \tilde{\eta}_{-r}^i) \right] 2 \sum_{w_\phi} (-1)^{w_\phi^7 + w_\phi^8 + w_\phi^9} |w_\phi^6, 2w_\phi^7, 2w_\phi^8, 2w_\phi^9, +\rangle_{RR}^{(0)}.
\end{aligned} \tag{4.10}$$

This expression shows that the twisted RR-sector in terms of (Y, ψ) is rewritten to have the same form as the untwisted RR-sector of D4-brane in terms of (ϕ, η) ²². If we use the basis (Y, ψ) for NSNS-sector and (ϕ, η) for RR-sector, each open string vacuum amplitude of NS-sector and R-sector cancels each other and the occurrence of the bose-fermi

²²This expression also implies that the original non-BPS D1-brane in T^4/\mathbf{Z}_2 can be thought as a BPS D4-brane in terms of the field (ϕ, η) with a “wrong GSO projection” $(-1)^F = I_4 \phi (-1)^{F\phi} = 1$, though the essence of this interpretation is not clear.

degeneracy is explicitly shown. Therefore we can say that the bosonization procedures at critical radius are crucial in the bose-fermi degeneracy.

5 Conclusions

In this paper we have shown explicitly in the boundary state formalism that the tachyon condensation in 2^{k-1} pieces of $D(p+2k) - \overline{D(p+2k)}$ at critical radii produces 2^{k-1} pieces of $Dp - \overline{Dp}$. Locally this means that a codimension $2k$ soliton of the tachyon field configuration corresponds to a Dp -brane (or \overline{Dp}). We have also verified this results in T^4/\mathbf{Z}_2 orbifold theory. Note that in these cases there are no gauge fields on the world volume. But the generations of lower D-brane charges indeed occur due to the Wess-Zumino terms which are peculiar to brane-antibrane systems [30]. In the boundary state description we have succeeded to see these phenomena explicitly.

In the process of the explicit calculations we have found two remarkable facts. The first is that the consistency with the open string picture (or Cardy's condition) requires the closed string sectors should have nontrivial Chan-Paton factors. This somewhat strange phenomenon only occurs if we discuss interactions of closed strings with brane-antibrane systems or non-BPS D-branes. These Chan-Paton factors also ensure the Wess-Zumino coupling proposed in [30].

The second one is the fact in the case of T^4/\mathbf{Z}_2 orbifold we can treat the twisted sector boundary state in the same way as the untwisted one by changing the field basis (or by "bosonization" procedure). This enables us to construct the boundary state which describe the tachyon condensation in the orbifold theory. Another application of this fact is the investigation of bose-fermi degeneracy [26, 27]. At the point where the degeneracy occurs the boundary state of a non-BPS D1-brane becomes very much like that of a BPS D-brane by using the bosonization procedure. Naively it seems that a sort of a symmetry is enhanced at this particular moduli, but it is difficult to see this explicitly even in our formalism. We leave this as a future problem. So far the tachyon condensation in four dimensional orbifold theories other than T^4/\mathbf{Z}_2 have not been discussed. If one try to construct the marginal deformations of BCFT in them, something like the previous bosonization procedures of the boundary state will be required.

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A Structure of boundary state

Here we give our CFT conventions and a short review of boundary states for general Dp-branes with or without the orbifold projection. Remember that we have used the light cone formulation [29, 13] and ignored²³ the non-zero modes of the fields $(Y^8, Y^9, \psi^8, \psi^9)$ in the case of $D2 - \overline{D2}$ system. We use almost the same conventions as Sen's except the detailed normalizations.

A.1 CFT conventions

We define $z = e^{-i\sigma_1 + \sigma_2}$ as the cylindrical coordinate of the world sheet and $w = \sigma_1 + i\sigma_2$ as its radial plane coordinate. First we list the mode expansions of (Y, ψ) fields :

$$\begin{aligned} Y_R^i(z) &= y_R^i - \frac{i}{2} p_{Y_R}^i \ln z + \frac{i}{\sqrt{2}} \sum_{n \neq 0} \frac{1}{n} \frac{(\alpha_Y)_n^i}{z^n}, \\ Y_L^i(\bar{z}) &= y_L^i - \frac{i}{2} p_{Y_L}^i \ln \bar{z} + \frac{i}{\sqrt{2}} \sum_{n \neq 0} \frac{1}{n} \frac{(\tilde{\alpha}_Y)_n^i}{\bar{z}^n}, \\ \psi_R^i(z) &= i^{\frac{1}{2}} \sum_{r \in \mathbf{Z} + \nu} \frac{\psi_r^i}{z^{r + \frac{1}{2}}}, \quad \psi_L^i(\bar{z}) = i^{-\frac{1}{2}} \sum_{r \in \mathbf{Z} + \nu} \frac{\tilde{\psi}_r^i}{\bar{z}^{r + \frac{1}{2}}}, \\ (\hat{p}_{Y_R}^i &= \sqrt{2}(\alpha_Y)_0^i, \quad \hat{p}_{Y_L}^i = \sqrt{2}(\tilde{\alpha}_Y)_0^i), \end{aligned} \tag{A.1}$$

where $\nu = \frac{1}{2}$ represents NS-sector and $\nu = 0$ R-sector and we set $i = 0 \sim 9$.

Then the OPE relations (2.6) are equivalent to the following (anti)commutation relations for modes :

$$\begin{aligned} [y_L^i, p_{Y_L}^j] &= [y_R^i, p_{Y_R}^j] = i\eta^{ij}, \\ [(\alpha_Y)_m^i, (\alpha_Y)_n^j] &= [(\tilde{\alpha}_Y)_m^i, (\tilde{\alpha}_Y)_n^j] = m\delta_{m, -n}\eta^{ij}, \\ \{\psi_r^i, \psi_s^j\} &= \{\tilde{\psi}_r^i, \tilde{\psi}_s^j\} = \delta_{r, -s}\eta^{ij}, \end{aligned} \tag{A.2}$$

where $\eta^{ij} = \delta^{ij}$ for $i = 1 \sim 9$ and $\eta^{ij} = -\delta^{ij}$ for $i = 0$. The vacuum of these modes is defined as $|\Omega\rangle_{NSNS}^{(0)}$. If we compactify the coordinates y^i on torus (radii R^i) then the momenta are quantized as follows

$$p_{Y_R}^i = \frac{n_Y^i}{R^i} + R^i w_Y^i, \quad p_{Y_L}^i = \frac{n_Y^i}{R^i} - R^i w_Y^i, \tag{A.3}$$

where n_Y^i and w_Y^i denote K.K. modes and winding modes.

We have also used the fields (X, χ) and (ϕ, η) as another bases. The mode expansions and commutation relations of these fields are defined in the same way.

²³In the case of the orbifold theory discussed in section 4, we ignored the non-zero modes of $(X^4, X^5, \psi^5, \psi^6)$.

A.2 Definition of boundary states

A boundary state of Dp-brane is defined by the following boundary conditions²⁴ in the closed string Hilbert space :

$$\begin{aligned}
\partial_2 Y^\mu(w)|_{\sigma_2=0}|Dp, \gamma \rangle &= 0, & (\mu = 0 \sim p), \\
\partial_1 Y^i(w)|_{\sigma_2=0}|Dp, \gamma \rangle &= 0, & (i = p+1 \sim 7), \\
(\psi_R^\mu(w) - i\gamma\psi_L^\mu(\bar{w}))|_{\sigma_2=0}|Dp, \gamma \rangle &= 0, & (\mu = 0 \sim p), \\
(\psi_R^i(w) + i\gamma\psi_L^i(\bar{w}))|_{\sigma_2=0}|Dp, \gamma \rangle &= 0, & (i = p+1 \sim 7),
\end{aligned} \tag{A.4}$$

where $\gamma = +, -$ is the spin structure on the boundary and the GSO projection of the closed string determines the correct linear combination of these spin structures. If we expand the left-hand side of eq. (A.4), we get

$$\begin{aligned}
((\alpha_Y)_n^\mu + (\tilde{\alpha}_Y)_{-n}^\mu)|Dp, \gamma \rangle &= 0, & (\mu = 0 \sim p), \\
((\alpha_Y)_n^i - (\tilde{\alpha}_Y)_{-n}^i)|Dp, \gamma \rangle &= 0, & (i = p+1 \sim 7), \\
(\psi_r^\mu - i\gamma\tilde{\psi}_{-r}^\mu)|Dp, \gamma \rangle &= 0, & (\mu = 0 \sim p), \\
(\psi_r^i + i\gamma\tilde{\psi}_{-r}^i)|Dp, \gamma \rangle &= 0, & (i = p+1 \sim 7).
\end{aligned} \tag{A.5}$$

These conditions are easy to solve by using the commutation relations (A.1),(A.2). Notice that for a BPS D-brane the boundary state consists of the NSNS-sector and RR-sector and the correct linear combination of them should be determined by comparing its cylinder amplitude with that of open string (see [22]). For example in the case of a (BPS) D2-brane the boundary state is given as eq.(3.1). Also note that for a non-BPS D-brane there is no RR-sector.

Finally let us see the orbifold case briefly. In general the orbifold theories have twisted sectors in the closed string Hilbert space and therefore it is necessary to add twisted sector boundary states $|T\rangle$ to the untwisted one. The twisted sector boundary states are defined by the same equation (A.4), but the mode expansion is different from (A.1) because of the twisted boundary condition. In the case of T^4/\mathbf{Z}_2 orbifold discussed in section 4, the mode expansion of (Y^i, ψ^i) ($i = 6, 7, 8, 9$) is shifted by half integer. For example, the twisted sector boundary state of $D0 - \overline{D0}$ is given as eq. (4.6), where we showed only the modes of (Y^6, ψ^6) . The correct linear combination of the twisted sector boundary states and the untwisted one is also determined by the calculations of the cylinder amplitude and this is called the Cardy's condition [23].

²⁴Of course the conditions remain the same if we replace (Y, ψ) with (X, χ) , because this procedure does not mix the Neumann and Dirichlet conditions.

B Equivalence of a boundary state and its bosonized version

In section 3, 4 we have used bosonized (and fermionized) descriptions of boundary states at special radii. In [24] the authors calculate several one point functions in the codimension one case and show that the results are the same as those before the bosonization. As a further evidence of the equivalence here we prove that the bosonized boundary states discussed in section 3.1 satisfy the correct boundary conditions in the case of the tachyon condensation in $D2 - \overline{D2}$ system. The other cases appeared in this paper can be treated almost in the same way.

B.1 Cocycle factors

In order to prove the correct boundary conditions the detailed cocycle factors should be given explicitly. For example, the fermionization relations (2.5) are written incorporating the cocycle factors as

$$\begin{aligned}\tau_i \otimes C_Y^i(\pm\sqrt{2}, 0) e^{\pm i\sqrt{2}Y_R^i}(z) &\cong \frac{1}{\sqrt{2}}(\xi_R^i \pm i\eta_R^i)(z), \\ \tau_i \otimes C_Y^i(0, \pm\sqrt{2}) e^{\pm i\sqrt{2}Y_L^i}(\bar{z}) &\cong \frac{1}{\sqrt{2}}(\xi_L^i \pm i\eta_L^i)(\bar{z}),\end{aligned}\tag{B.1}$$

where $\tau_i, C_Y^i(k_R, k_L)$ are both called cocycle factors. $\tau_i (i = 1, 2, 3)$ are 2×2 Pauli matrices. $C_Y^i(k_R, k_L)$ are defined by (for example see [33])

$$C_Y^i(k_R, k_L) \equiv \exp \left[\frac{1}{4} \pi i (k_{Y_R}^i - k_{Y_L}^i) (\hat{p}_{Y_R}^i + \hat{p}_{Y_L}^i) \right] \tag{B.2}$$

In bosonization procedure, they are needed to guarantee correct (anti)commutation relations between various fields.

Next step is the rebosonization of two fermions,

$$\begin{aligned}\frac{1}{\sqrt{2}}(\xi_R^i \pm i\psi_R^i)(z) &\cong \tilde{\tau}_i \otimes C_\phi^i(\pm\sqrt{2}, 0) e^{\pm i\sqrt{2}\phi_R^i}(z) \\ \frac{1}{\sqrt{2}}(\xi_L^i \pm i\psi_L^i)(\bar{z}) &\cong \tilde{\tau}_i \otimes C_\phi^i(0, \pm\sqrt{2}) e^{\pm i\sqrt{2}\phi_L^i}(\bar{z})\end{aligned}\tag{B.3}$$

In this way we accomplished changing variables from (Y^i, ψ^i) to (ϕ^i, η^i) (see Figure 3).

B.2 Proof of the correct boundary conditions

Now let us prove the facts that the bosonized boundary states satisfy the correct boundary conditions of the original ones and give a evidence that they are equivalent. We

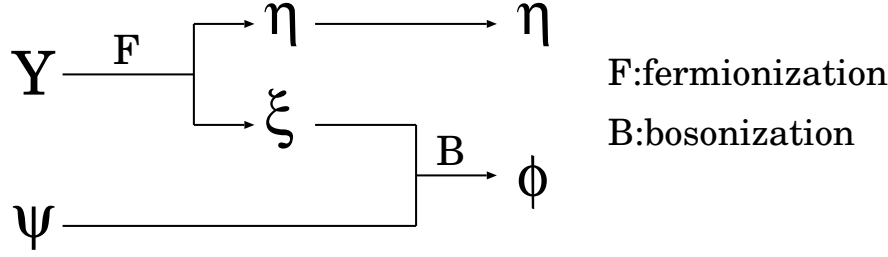


Figure 3: Flow of the bosonization

take the example of $D2 - \overline{D2}$ system discussed in section 3. Then two types of equivalence should be proved. The first is that eq.(3.12) and (3.13) are equivalent to eq.(3.10) and (3.11) respectively. We can verify this by showing that eq.(3.12) and (3.13) satisfy the boundary conditions eq.(3.14),(3.15). The second case is that $|B(\alpha = 1, \beta = 1), + \rangle$ (see eq.(3.23),(3.24)) is equivalent to the boundary state of $D0 - \overline{D0}$ system. We can also prove this in the same way by showing eq.(3.28),(3.29). Since these four equations can be proven in the same way, we show the proof of (3.14) below.

First let us note that eq.(3.12),(3.13) satisfy

$$(\eta_R^i(w) - i\eta_L^i(\bar{w}))|_{\sigma_2=0}|D2 - \overline{D2}, + \rangle_{NSNS,RR} = 0, \quad (B.4)$$

and that we can replace $\partial_2 Y^i(w)|_{\sigma_2=0}$ with (ϕ, η) variables. Then eq.(3.14) can be rewritten as

$$\begin{aligned} & \partial_2 Y^i(w)|_{\sigma_2=0}|D2 - \overline{D2}, + \rangle_{NSNS,RR} \\ &= \tilde{\tau}_i \otimes \frac{\sqrt{z}}{2i} \eta_R^i(z) \left[\sqrt{z} \left\{ C_\phi^i(\sqrt{2}, 0) e^{i\sqrt{2}\phi_R^i(z)} + C_\phi^i(-\sqrt{2}, 0) e^{-i\sqrt{2}\phi_R^i(z)} \right\} \right. \\ & \quad \left. + \sqrt{\bar{z}} \left\{ C_\phi^i(0, \sqrt{2}) e^{i\sqrt{2}\phi_L^i(\bar{z})} + C_\phi^i(0, -\sqrt{2}) e^{-i\sqrt{2}\phi_L^i(\bar{z})} \right\} \right] |D2 - \overline{D2}, + \rangle_{NSNS,RR}. \end{aligned} \quad (B.5)$$

The detail of the exponential is given as

$$\begin{aligned} & : e^{i\sqrt{2}\phi_R^i(z)} : |_{\sigma_2=0} \\ &= \exp[i\sqrt{2}\phi_R] \exp\left[-\frac{i}{\sqrt{2}}p_R\sigma_1\right] \exp\left[\sum_{n=1}^{\infty} \frac{1}{n}\phi_{-n}e^{-in\sigma_1}\right] \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n}\phi_n e^{in\sigma_1}\right]. \end{aligned} \quad (B.6)$$

If we note that eq.(3.12),(3.13) satisfy

$$(\phi_n + \tilde{\phi}_{-n})|D2 - \overline{D2}, + \rangle_{NSNS,RR} = 0, \quad (B.7)$$

then it is easy to see that the first and the third, the second and the fourth term in eq. (B.5) cancel respectively. The proof is almost the same as in the case of $|B(\alpha = 1, \beta = 1), + \rangle$ except that the \mathbf{Z}_2 -phases $(-1)^{w_\phi^1 + w_\phi^2}$ of eq. (3.27) play an important role for changing boundary conditions of Y^1, Y^2 .

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